Housing and Macroprudential Policy
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Abstract
This paper constructs a New Keynesian model with fully developed non-durable housing and financial sectors and a rich array of frictions. When shocked, house prices and production react violently, reflecting a notable characteristic of the recent Global Financial Crisis. A key aspect of the model is that non-durable consumption and housing both fall in response to an adverse monetary shock, demonstrating that the model overcomes the co-movement puzzle that hinders numerous other multi-sector models. Where collateral constraints amplify the initial effects of these shocks, it becomes important to find a policy rule that aids macroeconomic stability. This paper finds that a combination of monetary and macroprudential policy, focused on limiting the deviation of inflation, the output gap, house prices and loan extension from their steady state values can perform admirably in this regard in response to monetary shocks, but simpler Taylor Rules outperform when productivity shocks are the norm. While each of the rules considered are improvements over an inflation targeting regime, they come with a tradeoff. Households reliant on borrowing suffer negative changes in their welfare, even as those that save gain.

Keywords: borrowing constraints; banking; financial sector; housing; co-movement; optimal simple rules.
JEL Classification: E32; E50.

1 Introduction
For a long time, housing and financial sectors were, more often than not, omitted from macroeconomic models. The Global Financial Crisis of 2008 - which originated in the housing sector and was exacerbated by frictions in the financial sector - changed all that. Since then, there has been a scholarly rush to both develop new models and to retrofit the suite of existing models with the mechanisms necessary for the exploration of the broader implications of these sectors.

One of the more theoretically appealing of these papers was Monacelli (2009). This paper introduced collateral constraints into a model that featured durable good production as well as the archetypical non-durable production. However, the model in Monacelli

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1Durable good production is broadly analogous to housing production. Durable consumption and residential investment have been shown to have similar empirical properties in response to shocks and it can be argued that they have similar microfoundations. See Perks 2016a for a more in-depth explanation.
(2009) was not without its flaws. Crucially, it failed in its stated aim of resolving the durable good co-movement puzzle (Sterk 2011). Moreover it lacked financial intermediaries, making any deeper analysis of financial frictions and shocks impossible.

This paper resurrects the Monacelli (2009) model. By introducing nominal wage rigidities and a sophisticated banking sector, the model in this paper bring together the literature on durable goods, housing and financial frictions in an intuitive fashion and matches the key properties of the data. In combination, these mechanisms drive a model that, amongst other things, delivers substantial volatility in both house prices and production in response to a variety of shocks. This captures a key characteristic of the pre- and post Global Financial Crisis economy - house prices and production shoot up when economic conditions are unexpectedly good, but also plummet when they are worse than anticipated.

While it could be argued that it covers similar terrain to a number of other papers, the model presented here is unique. While the vast number of models in the financial frictions and intermediaries canon generally include either housing production or financial intermediaries, this one features both. As such, it sits alongside the work of Falagiarda and Saia (2013) and Brzoza-Brzezina, Kolasa and Makarski (2015) as one of the very few to feature a model with completely specified non-durable, housing and financial sectors as well as monetary and macroprudential policy. Having outlined a model capable of replicating the business cycle fluctuations of recent years, this paper asks how best to manage this volatility. Optimal simple monetary and macroprudential policy rules are calculated and used to contrast the relative effectiveness of various policy regimes in achieving macroeconomic stability, as well as the tradeoffs they imply.

The analysis in this paper reveals that there may be a role for macroprudential policy, or even monetary policy that leans against increases in house prices, to combat the effects of various shocks, particularly in the case of an unanticipated increase in the policy rate. In the event of a technology shock, however, the rules that incorporates house prices are inferior, and macroprudential policy has no positive effect. All of the rules are relatively ineffective in combating a surprise hike in the borrowing rate. This suggests that the best option for the policymaker is contingent on the origin of the shocks hitting the economy.

The adoption of these rules, however, does not yield a pareto gain. Social welfare is improved, but while household savers enjoy greater consumption after the switch from an inflation targeting regime to a simple or augmented Taylor Rule (with or without macroprudential policy), household borrowers receive less.

The paper is structured as follows. Section 2 offers a brief literature review. Section 3 describes the model. Section 4 examines the parameterisation and calibration of the model. Section 5 presents the key results, before Section 6 examines the welfare effects of various policy rules and section 7 concludes.

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2This is because, as discussed in more detail subsequently, a substantial proportion of these models build on the model in Iacoviello (2005) which assumes housing is in fixed supply. These models have found this simplifying assumption to be sufficient for their research purposes, and see few qualms in omitting residential investment, despite its strong procyclical routinely being cited as a characteristic of the business cycle (Alvarez and Cabrero 2010).

3To demonstrate this, Appendix A includes a list of comparable models and categorises their features.
2 Literature Review

This paper lives at the intersection of several avenues of the business cycle literature. It brings together and builds on the efforts and insights of the research into durable goods, housing, borrowing constraints, financial intermediaries and macroprudential policy.

The durable good literature was shaped by the seminal work of Barsky, House and Kimball (2007). Their paper - the first elucidation of the durable goods co-movement puzzle - showed how hard it was to construct a simple multi-sector New Keynesian model that remained faithful to the microevidence on the frequency of price changes and reflected key properties of the data. While empirical analyses suggest that durable and non-durable consumption co-move positively, early two sector models featuring a flexibly priced durables sector demonstrated negative co-movement in the wake of a monetary policy shock. Put even more bluntly, despite the data suggesting a fall in both durable and non-durable consumption after a monetary policy contraction, these models tended to suggest that durable consumption would increase even as non-durable consumption decreased.

Their work engendered substantial research aimed at resolving the puzzle. To date, suggested solutions have included real (Di Pace and Hertweck 2012 and Cenesiz and Guimaraes, 2013) and nominal wage rigidities (Di Cecio, 2009 and Carlstrom and Fuest, 2010), time-varying markups (Katayama and Kim 2010 and Perks 2016b) and inter-sectoral linkages (Bouakez, Cardia and Ruge-Murcia 2011; Petrella, Rossi and Santoro 2012, Sudo, 2012 and Perks 2016a). Some of these have proved more effective (or plausible) than others.

For his part, Monacelli (2009) proposed the incorporation of household borrowers and lenders who traded in a debt market subject to a collateral constraint into an otherwise standard New Keynesian model. Monacelli (2009) argued that these innovations would break the quasi-constancy of the shadow value of durable goods, leading it to fluctuate more in response to the shock and, thus, go some way towards resolving the puzzle. While it did capture a couple of nice features of the economy, a subsequent paper by Sterk (2011) showed that Monacelli (2009)’s setup actually made the problem more acute. It took the introduction of capital (by Chen and Liao, 2014) to reconcile the model with its stated aim.

By way of contrast, the housing literature has barely acknowledged the co-movement puzzle (despite the fact that it applies equally to residential investment (Aoki, Proudman and Vliegh 2004 and Cantelmo and Molina, 2015). Rather than model the production of housing, many papers (including the one closest in spirit to the model in this paper - Gerali, Neri, Sessa and Signoretti, 2010) have followed the lead of Iacoviello (2005) in assuming that housing is exogenously given and in fixed supply. The reason for this is a desire to marry simplicity with necessity - without overly complicating the model, a fixed supply of housing offers a collateral stock against which agents can borrow.

Collateral constraints have a long and rich history in macroeconomic theory. Their theoretical foundations can be traced back at least as far as Kiyotaki and Moore (1997), who used land as a collateralizable asset to facilitate borrowing between patient and impa-
tient households. In their model, decreases in the value of land have an adverse affect on the impatient household’s ability to borrow via a collateral constraint. This further compounds the slump in land prices and results in an amplification of the initial shock. Iacoviello (2005) built upon this by incorporating the smaller theoretical model into a New Keynesian model\(^4\). A battery of papers have since used this framework to address a myriad of research questions, from the wealth effects of the houseing market (Iacoviello and Neri 2009) to optimal macroprudential policy (Rubio and Carrasco-Gallego 2015) and many others besides, with at least one reason for its popularity being the ease with which a financial sector can be interposed between the households in the debt market.

Given the crucial role of debt markets and collateral constraints in amplifying shocks (and the dominant role banks have in this market), a growing number of papers have sought to examine the effects of financial sector frictions. Goodfriend and McCallum (2007) explicitly incorporated a banking sector into the financial accelerator model of Bernanke, Gertler and Gilchrist (1999) in order to describe the interaction and differences between various types of interest rates to determine how much the central bank may be misled by relying on the standard model. This was quickly followed by Curdia and Woodford (2009), which emphasised debt contracts between households, rather than between households and the firm. Gerali, Neri, Sessa and Signoretti (2010) went even further by including an imperfectly competitive banking sector in both the deposit market and the loan market and showing that the monopolistic power of banks over the loan and deposit rate changes the pass-through of the policy rate. This setup has important consequences for monetary policy because the central bank policy rate is not transmitted fully and instantaneously into households and firms decisions.

The incorporation of these mechanisms also opened up new paths for policy research. Whereas, in the simplest New Keynesian models, adjustment of the nominal interest rate via a Taylor-type Rule was effectively the only policy choice, more complex models with collateral constraints and financial intermediaries admitted a role for macroprudential policy. The presence of loan to value ratios that regulated the proportion of collateral that could be borrowed against and capital adequacy ratios that restricted a financial intermediary’s ability to extend loans ensured that policy makers had more tools in their locker, and were faced with more choices, when deciding how best to achieve their macroeconomic policy objectives.

3 The Model

This model is a bridge between the durable goods and housing literatures. It builds nominal wage rigidities and a rich financial sector into the model proposed by Monacelli (2009), while it distinguishes itself from models in the vein of Iacoviello (2005) and Gerali, Neri, Sessa and Signoretti (2010) by explicitly modeling housing and abstracting from capital production.

The introduction of nominal rigidities plays a key role in ensuring the the non-durable and housing sectors move in similar directions in response to a monetary policy shock. Since a monetary policy shock only has real (and direct) effects on the sector with sticky

\(^4\)Loans in the early financial accelerator models of Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1999) differed in that they were were allocated via an external finance premium.
prices, an unanticipated contractionary shock would cause production of non-durable goods to fall and less labour to be employed. If wages were able to freely adjust, firms in the flexibly priced durable goods sector would capitalise by procuring more labour and expanding. By making it more difficult for wages to adjust, nominal wage rigidities disrupt the flow of labour between the sectors and help to ensure positive co-movement.

The more substantial difference is the incorporation of a financial sector. The establishment of an imperfectly competitive financial sector disconnects the activities of borrowers and savers, and creates a profit wedge for the financial intermediaries to exploit. Not only does this mirror the activity of banks in the real economy, but it also allows analysis of financial frictions and shocks originating in or hitting this part of the model economy. It also further embeds macroprudential policy in the model and enriches it as a tool for the pursuit of macrostability.

Otherwise, the skeleton of this model is very similar to that proposed by Monacelli (2009). At its core, it is a two sector New Keynesian model with nominal price rigidities. These sectors differ in that one produces non-durable consumption goods with a modicum of price stickiness while the other adds to the existing stock of housing - a stock that depreciates slowly over time - and is characterised by flexible prices.\footnote{In a study on goods prices, Klenow and Malin 2010 found that the more durable the good, the more flexible the price. This accords with the argument of Carlström and Fuerst, 2010 that there really is no justification for houses - which are traded after independent negotiations - to have anything other than flexible prices.}

The model economy is populated by six distinct agents. These are:

- household savers
- household borrowers
- final good producers
- intermediate good producers
- financial intermediaries
- a monetary and macroprudential authority.

In any given period, household savers sell their labour to intermediate goods producers at a mark-up over their marginal product of labour (reflecting the differentiated labour each household supplies in the imperfectly competitive labour market). This income, along with the profits from durable and non-durable producers and the financial intermediaries (which they own by assumption) and the return on deposits with the financial intermediary in the previous period are spent on non-durable goods and housing, and deposited as savings for the next period.

Household borrowers discount the future more heavily than savers. They also sell their labour to intermediate good producers at a mark-up and borrow from financial intermediaries in order to purchase non-durable goods and housing and repay loans from the preceding period. Since the size of the population is normalised to one, the proportion of borrowers in the economy is $\omega$, while the proportion of the savers in the economy is $1 - \omega$.\footnote{In a study on goods prices, Klenow and Malin 2010 found that the more durable the good, the more flexible the price. This accords with the argument of Carlström and Fuerst, 2010 that there really is no justification for houses - which are traded after independent negotiations - to have anything other than flexible prices.}
Financial intermediaries sit between household borrowers and savers. They collect deposits from savers, deposit them with the monetary authority and then pay them out in the following period at a markdown on the policy rate. They also strike loan contracts with the borrowers, offering them funds in the current period (which they themselves procure from the monetary authority) to be repaid in the next period at a mark-up over the policy rate.

Unlike the original Monacelli (2009) model, household borrowers and savers do not trade debt contracts directly. Because the monetary authority provides liquidity to the financial intermediaries (in addition to conducting monetary policy according to a simple Taylor Rule), the deposits of savers do not need to equal the loans of borrowers and the policy rate remains at the heart of all financial transactions.

Intermediate goods producers are monopolistically competitive and face nominal price rigidities. They hire household borrowers and savers (at a markup over their marginal product), set the price of their output (if permitted to do so) and sell their output to final goods producers in order to maximise profits. As noted previously, these profits are returned to the household savers. Having purchased the output of the intermediate goods producers, final goods producers aggregate it into final goods which are then sold back to the households. Since it is assumed that there is a unit mass of these, they are perfectly competitive. As such, they earn no profits.

3.1 Households

3.1.1 Household Borrowers

Household borrowers gain utility from consumption \(X^b_t\) and disutility from labour \(N^b_t\). Their problem is to maximise their discounted lifetime utility, given by:

\[
U_t = E_t \left\{ \sum_{t=0}^{\infty} \beta^t U_t(X^b_t, N^b_t) \right\}. \tag{1}
\]

The rate at which they discount future periods is given by \(\beta_t\). \(X^b_t\) is an index comprised of final non-durable goods \(C^b_t\) and housing \(H^b_t\). This is given by:

\[
X^b_t \equiv \left[ (1 - \alpha)^{\frac{1}{\tau}} C^b_t + \alpha^\frac{1}{\tau} H^b_t \right]^{\frac{\eta}{\eta}}. \tag{2}
\]

Consumption of \(C^b_t\) and \(H^b_t\) is at the end of period \(t\). \(H^b_t\) comprises purchases made within the period and, because housing lasts for a number of periods, any stock left from the previous period. \(\alpha > 0\) is the share of durable goods in consumption and \(\eta \geq 0\) is the elasticity of substitution between non-durable and durable goods.

Borrowers also face a budget constraint and a borrowing constraint. The budget constraint, in nominal terms takes the form:

\[
P_{c,t} C^b_t + P_{h,t} \left( H^b_t - (1 - \delta) H^b_{t-1} \right) + r^b_{t-1} B_{t-1} = B_t + W^b_t N^b_t. \tag{3}
\]

\(^6\)Where variables and parameters that can be similarly interpreted appear in the problems of the borrowers and savers, they are denoted with either a \(b\) or an \(s\).
\( P_{c,t} \) and \( P_{h,t} \) are the prices of non-durable goods and housing in period \( t \) respectively. \( B_t \) is the nominal debt, or the amount that the household has agreed to borrow from the financial intermediary, as at the end of period \( t \). \( r_{t-1}^{b} \) is the nominal rate on loan contracts agreed at \( t - 1 \). \( \delta \) is rate at which housing depreciates. \( W_t^b \) is the nominal wage rate chosen by the household (discussed in more detail in sub-section 3.4.2.)

In real terms, the budget constraint becomes:

\[
C_t^b + q_t \left( H_t^b - (1 - \delta)H_{t-1}^b \right) + r_{t-1}^{b} \frac{b_{t-1}}{\pi_{c,t}} = b_t + \frac{W_t^b}{P_{c,t}} N_t^b, \tag{4}
\]

where \( b_t \equiv \frac{B_t}{P_{c,t}} \), \( \pi_{c,t} = \frac{P_{c,t}}{P_{c,t-1}} \) and \( q_t \equiv \frac{P_{h,t}}{P_{c,t}} \), or the relative price of housing.

The borrowers also face a borrowing constraint. The amount that the borrower will have to repay tomorrow can not exceed the expected value of the agent’s durable stock tomorrow after depreciation. This borrowing constraint can be expressed as:

\[
r_t^b B_t \leq (\chi_t) \left( 1 - \delta \right) E_t \left\{ \frac{H_t^b P_{h,t+1}}{r_t^b} \right\} \tag{5}
\]

\( \chi_t \) is the proportion of the housing stock which can be collateralised. Since it effectively caps the size of the loan, it works in much the same way as a loan to value ratio. As such, it is one of the tools at the disposal of a macroprudential regulator.

In the neighborhood of the deterministic steady state, it is assumed that the borrowing constraint always binds. In real terms, it becomes:

\[
b_t = (\chi_t) \left( 1 - \delta \right) E_t \left\{ \frac{q_{t+1}H_t^b}{r_t^b} \frac{1}{\pi_{c,t+1}} \right\} \tag{6}
\]

Thus, the problem of the household borrower is to maximise (1) subject to (4) and (6).

### 3.1.2 Household Savers

Household savers are more patient than household borrowers. Because they don’t discount the future as much, their discount factor \((\beta^s)\) is greater than that of household borrowers \((\beta^b)\). Household savers are also assumed own both the intermediate goods producers and the financial intermediaries. The profits they receive effectively augments their income.

Similar to the borrowers, savers derive utility from consumption and disutility from labour.

\[
\bar{U}_t = E_t \left\{ \sum_{t=0}^{\infty} \beta_t^s U \left( X_t^s, N_t^s \right) \right\}. \tag{7}
\]

The consumption index \((X_t^s)\) is given by:

\[
X_t^s \equiv \left[ (1 - \alpha) \frac{1}{\gamma} \left( C_t^s \right)^{\frac{n-1}{\gamma}} + \alpha \frac{1}{\gamma} H_t^s \frac{n-1}{\gamma} \right]^{\frac{\gamma}{n-1}}. \tag{8}
\]
Household savers maximise their utility subject to a single constraint. This budget constraint is similar to that of the borrower, with a couple of exceptions. Savers transfer consumption into future periods by making deposits \(d_t\) with the financial intermediary, while deposits from the previous period are returned at the nominal rate \(r^d_{t-1}\) greed in the previous period. They also receive profits from the production of intermediate goods \((\Gamma_{j,t} \text{ where } j \in \{c, d\})\) and from the activities of financial intermediaries \((J_t)\). In nominal terms, this is given by:

\[
P_{c,t}C^s_t + P_{d,t} \left( H^s_t - (1 - \delta) H^s_{t-1} \right) \equiv D_t + W^s_t N^s_t + r^d_{t-1} D_{t-1} + \frac{1}{1 - \omega} \left( \sum_{c \in h} (P_{j,t} \Gamma_{j,t} + J_t) \right)
\]

In real terms, the budget constraint is given by:

\[
C^s_t + q_t \left( H^s_t - (1 - \delta) H^s_{t-1} \right) \equiv d_t = \frac{W^s_t}{P_{c,t}} N^s_t + r^d_{t-1} \frac{d_{t-1}}{\pi_{c,t}} + \frac{1}{1 - \omega} \left( \Gamma_{c,t} + q_t \Gamma_{h,t} + (1 - \omega) J_t \right)
\]

where \(d_t \equiv \frac{D_t}{P_{c,t}}\).

As a result, the problem of the household borrower is to maximise (7) subject to (10).

### 3.2 The Financial Sector

Banks play an important role in the model. They intermediate all financial transactions by collecting deposits from household savers and offering loans to household borrowers. Striking a contract with a bank is the only way for households to either delay or bring forward their consumption. While there are some minor differences, the establishment of the financial sector in this paper largely follows Gerali, Neri, Sessa and Signoretti (2010).

For their role to be both meaningful (in a modeling sense) and reflective of the broader economy, there needs to be some frictions in the financial sector. These frictions are introduced via monopolistic competition and bank capital.

The presence of monopolistic competition gives banks market power. This allows them to adjust rates on loans and deposits in response to shocks and other changes in the economy. The degree to which a bank passes on changes in the deposit and loan rates can vary depending on conditions in the economy. Not only does this offer a justification for the profits commonly seen in the sector (via the wedge between deposit and lender rates), but it also provides a channel through which bank capital can transmit shocks through the economy.

Bank capital is facilitated by assuming that banks must obey a balance sheet identity (11). This identity captures the notion that all funds lent by a bank must be financed by either bank capital or deposits. As sources of capital, these two are perfect substitutes. In order to pin down how the bank makes its financing decision, it is assumed that there is an exogenously given target capital to assets ratio that holds with equality. This target is analogous to a capital adequacy ratio and offers a second macroprudential policy instrument (in addition to the loan to collateral ratio discussed previously) for analysis. Setting up the model in this way makes bank capital integral to the model. Because it is
accumulated out of retained earnings, it links the real and financial sides of the economy and has implications for the business cycle. For example, in a downturn, falling bank profits restrict the accumulation of capital and thus limit the bank’s ability to write new loan contracts, further exacerbating the short term contraction.

To incorporate this in a tractable fashion into the model, there is a unit mass of atomistic banks indexed by $m$. Each bank has two sides with distinct functions. These can be thought of as wholesale branches and retail branches.

### 3.2.1 Wholesale Branches

The wholesale side of the bank is interposed between the retail side (which interacts directly with the households) and the central bank in the interbank market. The wholesalers provide loans to retailers ($b_{m,t}$) at a rate of $R_b^d$ who then offer them to impatient households at a markup. They also collect deposits from retailers ($d_{m,t}$) and pay them at the rate offered by the central bank ($R_b^d = R_t$).

Since a bank’s liabilities must always equal its assets, the wholesale side of the $m$th bank combines bank capital ($k_{m,t}$) and wholesale deposits ($d_{m,t}$) to fund wholesale loans ($b_{m,t}$). This can be written as:

$$b_{m,t} = d_{m,t} + k_{m,t} \quad (11)$$

Banks are also subject to a capital adequacy target

$$k_{m,t} = \frac{b_{m,t}}{\eta}. \quad (12)$$

$\eta$ is broadly equivalent to an exogenously imposed capital adequacy requirement. When the bank deviates from this, it incurs quadratic costs.

From this it follows that if the bank wants to increase the amount of loans it offers, it must accumulate bank capital. It can do this by adding some of the profits of its activities to the undepreciated stock of capital held over from the previous period. This is given by:

$$k_{m,t} = (1 - \delta^f)k_{m,t-1} + w^f j_{m,t-1} \quad (13)$$

The rest of the profits, or $(1 - w^f)j_{m,t-1}$ are returned to patient households, who are assumed to own the banks.

The wholesale branch’s dividend policy is assumed to be exogenous. Thus, wholesale branches cannot choose their level of bank capital. Instead they choose loans ($b_{m,t}$) and deposits ($d_{m,t}$) to maximise profits ($j_{m,t}$). Or:

$$\max_{b_{m,t},d_{m,t}} E_0 \sum_{t=0}^{\infty} \beta^t \left[ R_b^d b_{m,t} - R_b^d d_{m,t} - k_{m,t} \right] - \frac{\kappa}{2} \left( \frac{k_{m,t}}{b_{m,t}} - \eta \right) k_{m,t} \quad (14)$$

subject to:

$$b_{m,t} = d_{m,t} + k_{m,t}. \quad (15)$$
$R^b_t$ and $R^d_t$ - the gross wholesale loan and deposit rates respectively - are taken as given.

The first order conditions of the wholesale branches profit maximisation problem yields a condition that links the spread between the wholesale banks loan and deposit rates to the leverage $(b_t/k_t)$ of bank $m$. Formally, this is given by:

$$R^b_t = R^d_t - \kappa \left( \frac{k^t_{m,t}}{k_{m,t}} - \eta \right) \left( \frac{k^t_{m,t}}{k_{m,t}} \right)^2,$$

(16)

Since the wholesale branches invest any excess funds they receive with the central bank at the riskless rate ($R_t$), then $R^d_t = R_t$ in the interbank market. Moreover, as the interbank market is populated by a unit mass of identical wholesale banks, (16) can be re-written as:

$$R^b_t = R_t - \kappa \left( \frac{k^t_{m,t}}{k_{m,t}} - \eta \right) \left( \frac{k^t_{m,t}}{k_{m,t}} \right)^2.$$

(17)

This equation highlights the role of capital in determining the wholesale banks willingness to supply loans. Since there is a wedge between the loan and the policy rate, the wholesale bank would like to extend as many loans as possible. However, when leverage increases, the capital to asset ratio moves away from $\eta$ and banks incur costs that mitigate these profits. Thus the optimal choice for banks is to choose a level of loans that equates the marginal cost of adjusting the capital to asset ratio and the deposit-loan spread.

### 3.2.2 Retail Branches

Retail branches interact with households in markets for deposits and loans. It is assumed that the markets in which the retail branches operate are monopolistically competitive. This gives them some power to set both deposit and loan rates. Once set, however, the presence of price adjustment costs means that the retailers do not have the ability to freely adjust them.

In the market for loans, retail branches borrow $b_{m,t}$ from the wholesale branch at the rate $R^b_t$, differentiate them at no cost and then offer these funds to households borrowers at a mark-up. The problem of the retail branch, as it applies to loans, is to choose the return on loans made to household borrowers ($r^b_t$) to maximise profits ($j^t_{r,t}$). This is given by:

$$\max_{r^b_{m,t}} E_0 \sum_{t=0}^{\infty} \left\{ b^t_{m,t} b_{m,t} - R^b_t b_{m,t} - \frac{\kappa^l}{2} \left( \frac{r^b_{m,t}}{r^b_{m,t-1}} - 1 \right) r^b_t b_t \right\},$$

(18)

subject to demand for loans, or:

$$b_{m,t} = \left( \frac{r^b_{m,t}}{r^b_t} \right)^{-\epsilon^l} b_t,$$

(19)

where $\kappa^l$ represents the cost of adjusting the loan rate.

As in Gerali, Neri, Sessa and Signoretti (2010), assuming a symmetric equilibrium, the first order conditions of this problem yield a loan rate setting equation given by:
When log-linearised, this becomes:

\[
\frac{\kappa^d}{\epsilon^d + 1} = \frac{\kappa^d}{\epsilon^d + 1} + \beta^d E_t \left\{ \frac{\lambda^{d+1}}{\lambda^d} \frac{r_t}{r_t - 1} - 1 \right\} \frac{d_{t+1}}{d_t} \right\} = 0. \tag{20}
\]

From this it can be seen that the repayment rate on loans depends on a number of factors. Not only does it depend on competition in the loan market (\(\epsilon^l\)) and the costs of adjusting the loan rate (\(\kappa^l\)), but it also depends on the expected future path of the wholesale loan rate, which, as shown in the preceding sub-section, is linked to the policy rate and the bank’s capital position.

If it is assumed that there is no cost associated with adjusting retail loan rates, \(\tag{20}\) can be simplified to:

\[
\frac{\kappa^d}{\epsilon^d + 1} - \frac{\kappa^d}{\epsilon^d + 1} - \beta^d E_t \left\{ \frac{\lambda^{d+1}}{\lambda^d} \frac{r_t}{r_t - 1} - 1 \right\} \frac{d_{t+1}}{d_t} \right\} = 0. \tag{20}
\]

The problem for the retail branch in the market for deposits is the opposite of that in the market for loans. The retail branches collect \(d_{m,t}\) from the household savers. The return on storing these funds with the wholesale branch for a period is the policy rate (\(R_t\)). This rate is offered to household savers at a markdown (reflecting monopolistic competition for deposit services). The problem of the retail branch is to choose the deposit rate (\(r^d_t\)) it will offer to household borrowers in order to maximise profits (\(j^d_{m,t}\)). Or:

\[
\max_{r^d_{m,t}} E_0 \sum_{t=0}^{\infty} \left\{ R_t d_{m,t} - r^d_{m,t} - \frac{\kappa^d}{2} \frac{r^d_{m,t}}{r^d_{m,t - 1}} - 1 \right\} \frac{d^d_{m,t}}{d_t} \right\} = 0. \tag{23}
\]

subject to deposit demand of:

\[
d_{m,t} = \frac{r^d_{m,t}}{r^d_t} \right\} = 0. \tag{24}
\]

where \(\kappa^d\) represents the cost of adjusting the deposit rate once set. If a symmetric equilibrium is assumed, the first order conditions of this problem yield a deposit rate setting equation of:

\[
0 = -1 + \epsilon^d_t + \epsilon^d_t \frac{R_t}{r^d_t} - \kappa^d \frac{r^d_t}{r^d_{t - 1}} - 1 \right\} \frac{d^d_{t+1}}{d_t} \right\} = 0. \tag{25}
\]

When log-linearised, this becomes:

\[
\frac{\kappa^d}{\epsilon^d_t + 1 + (1 + \beta^d)\kappa^d} \frac{r^d_t}{r^d_{t - 1}} \right\} \frac{d^d_t}{d_{t+1}} \right\} = 0. \tag{26}
\]
The rate at which deposits are paid to household savers depends on the expected future level of the policy rate. How quickly the retail branch adjusts to changes in the policy rate depends on the magnitude of the adjustment costs \((\kappa^d)\) and the degree of competition in the banking sector \((\kappa^d)\).

Similar to retail loan rates, if it is assumed that there is no cost to adjusting retail deposit rates, \((25)\) can be simplified to:

\[
r_t^d = \frac{\epsilon^d}{\epsilon^d - 1} R_t
\]

(27)

where \(\epsilon^d < 0\) reflects the markdown applied to the policy rate.

The financial sector shock is defined as an unanticipated increase in the household’s borrowing rate. This is akin to the resetting of adjustable mortgage rates during the early months of the Global Financial Crisis. Using \(L_t\) to represent this shock, \((22)\) can be rewritten as:

\[
r_t^b = \begin{bmatrix} \epsilon^l \\ \epsilon^l - 1 \end{bmatrix} R_t L_t
\]

(28)

where

\[
\log \epsilon_{l,t} = \rho_L \log \epsilon_{l,t-1} + \epsilon_{l,t},
\]

(29)

with \(\rho_L < 1\) and \(\epsilon_{l,t} \sim i.i.d\.

### 3.3 Final Good Producers

There is a single final goods producers in both the non-durable goods and housing sectors. (Because the problem is the same in each sector, the notation is simplified by removing \(j \in (c, h)\) from all variables.) Each final goods producer purchases slightly differentiated inputs from a continuum of intermediate goods producers populated on the unit interval (and indexed by \(i\)) and aggregate them according to:

\[
Y_t = \int_{j}^{j_{i+1}} \frac{Y_{t,j_{i+1}^{-1}}^{-1}}{\epsilon_{j_{i+1}^{-1}}} \, di.
\]

(30)

Their problem is to choose the amount of intermediate goods to purchase and transform them into final goods in order to maximise profits. Their objective is to maximise:

\[
P_t Y_t - \int_{t}^{t+1} P_{t,i} Y_{i,t} di,
\]

(31)

subject to \((30)\).

Reflecting its monopolistic nature and the market power of intermediate goods producers, final goods producers take the price of the inputs they purchase as given. It is assumed that prices are aggregated according to:

\[
P_t = \int_{j}^{j_{i+1}} P_{t,j}^{1-\epsilon_{j_{i+1}^{-1}}} \, di.
\]

(32)
Solving this maximisation problem yields the optimal demand for the intermediate good supplied by the ith producer. This demand is given by:

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon_j} Y_t.$$  \hspace{1cm} (33)

### 3.4 Intermediate Good Producers


The first difference is the presence of nominal wage rigidities. Absent from the Monacelli (2009) model, wage stickiness is key to reconciling the model with the data. By slowing the response of wages to external shocks, nominal wage rigidities impede the flow of labour across sectors and help to ensure that a contraction in one doesn’t automatically imply an expansion of the other.

The second difference is that nominal price rigidities are introduced ala Calvo (1983), rather than the Rotemberg (1982) setup used in the Monacelli (2009) paper. Since the Calvo (1983) and Rotemberg (1982) constructs have been shown to be qualitatively and quantitatively similar in many regards, this departure is not particularly significant.

As with final goods producers, the notation is simplified by omitting $j \epsilon(c,h)$ from all variables.

#### 3.4.1 Nominal Wage Rigidities

Intermediate goods producers hire labour from households and use it to produce intermediate non-durable and housing goods according to the linear production function:

$$Y_{i,t} = A_t N_{i,t}.$$  \hspace{1cm} (34)

$A_t$ represents an aggregate technology or productivity shock. This shock follows an exogenous AR(1) process given by:

$$\log \varepsilon_{a,t} = \rho_a \log \varepsilon_{a,t-1} + \Upsilon_{a,t},$$  \hspace{1cm} (35)

with $\rho_a < 1$ and $\Upsilon_{a,t} \sim i.i.d$.

Each household offers a slightly differentiated type of labour. This assumption, across the unit mass of households, implies that workers can be indexed by $k \in [0,1]$.

Total labour used by firm $i$ is given by:

$$N_{i,t} = \left(\int_0^1 N_{i,k,t} \frac{\varepsilon_w^{-1}}{\varepsilon_w} dk\right)^{\varepsilon_w^{-1}}$$  \hspace{1cm} (36)

where the elasticity of substitution between household labor types is given by $\epsilon_w$. As long as $\epsilon_w << \infty$, household labour types are imperfect substitutes.
Intermediate goods producers problem is to find the output maximizing combination of labor for any given level of labor costs. This results in demand by firm $i$ for labour type $k$ being given by:
\[
N_{i,k,t} = \left( \frac{W_{k,t}}{W_t} \right)^{-\epsilon_w} N_{i,t}. \tag{37}
\]
where:
\[
W_t = \left( \int W_{k,t}^{1-\epsilon_w} dk \right)^{\frac{1}{1-\epsilon_w}}. \tag{38}
\]
The choice of wage by the households rests on the extent to which there are close substitutes for the labour they offer (reflected in $\epsilon_w > 1$) and how frequently they are able to adjust their wages. In any given period, a constant fraction of households $(1 - \theta^w)$ are able to do so, while the rest $(\theta^w)$ retain the wage from the previous period. The household problem, in this regard, is to choose $W_{k,t}$ to maximise household utility subject to demand for their labour \[37\] and their budget constraint, given that their choice may prevail well into future periods.

The first order condition of this problem gives the optimal wage equation. When this is log-linearised around the steady state, it yields an aggregate New Keynesian Wage Phillips Curve for each type of household. With $t \in (s,b)$, these can be written as:
\[
\hat{\pi}_{t}^{w,l} = \beta^l E_t \hat{\pi}_{t+1}^{w,l} - \lambda^{w,l} W_t^{l-\epsilon_w} \mu_t^{w,l} \tag{39}
\]
where $\hat{\pi}_{t}^{w,l}$ denotes $\frac{W_{l,t}}{W_{l-1,t}}$ and $\mu_t^{w,l}$ is the log-difference from each household’s steady state mark-up of wage over the marginal rate of substitution between consumption and labour, or the wage that would prevail were markets perfectly competitive and $\lambda^{w,l}$ represents $(1-\theta^w)(1-\theta^w)^{-1}(1+\epsilon_w-\theta^w)$.  

The intuition for this is simple. When the average wage in the economy is below the level required to maintain the desired markup (on average), households that are able to adjust their wage choose to increase it. This creates wage inflation.

### 3.4.2 Nominal Price Rigidities

Each individual intermediate goods producer produces its own slightly differentiated good. The market for this good is monopolistically competitive, reflecting the lack of perfect substitutes (captured by $\epsilon_j$). As a result, each firm $i$ is able to set the price for its differentiated product. Once chosen, these prices cannot be freely adjusted period to period. Rather, only a constant proportion of these firms (given by $1 - \theta$) will be randomly selected to change their price in any given period. This is akin to $P_t = P_{t-1}$ with probability $\theta$.

The problem of the intermediate producer is to choose the price that maximises profits given demand for their particular product \[33\], their production function \[34\], the wage set by the households and demand for each type of labour (represented in the preceding subsection by \[37\]).
The first order condition of this problem yields the optimal real price. After log-linearisation around the steady state, a New Keynesian Phillips Curve for each sector can be derived, given by:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa_j \hat{m}c_t$$

(40)

where $\hat{m}c_t$ is the percentage deviation of marginal cost in period $t$ from its steady state value and $\kappa_j = \frac{(1-\theta_j)(1-\theta_j^*)}{\theta_j}$.

When marginal cost is anticipated to be higher than its steady state value, the intermediate goods producers that are able to change their price adjust it above the average price level in order to maintain their desired markup over marginal cost. This drives price inflation in the model economy.

Where prices are perfectly flexible - as is the case in the housing sector - intermediate goods producers are able to adjust their price in every period. They adjust their prices to maintain a constant mark-up over marginal cost. This is given by:

$$P_{h,t} = \frac{\epsilon}{\epsilon - 1} mc_t$$

(41)

### 3.5 Monetary and Macroprudential Policy

Monetary policy is conducted according to a Taylor Rule that targets punishes the deviation of growth, output and house prices from their steady state values. This rule is given by:

$$R_t = \left( \frac{yt}{y} \right)^{\phi_y} \left( \frac{\pi_t}{\pi} \right)^{\phi_\pi} \left( \frac{q_t}{q} \right)^{\phi_q} \epsilon_t,$$

(42)

where $\pi_t = \pi^{1-\alpha}_t, \pi^{\alpha}_{d,t}$, $yt = y^{1-\alpha}_t, y^{\alpha}_{d,t}$ and $\phi_{\pi} > 1$ in order to adhere to the Taylor Principle. Similar to the financial sector and technology shocks, the monetary shock is assumed to follow an exogenous AR(1) process which is given by:

$$\log \varepsilon_{m,t} = \rho_m \log \varepsilon_{m,t-1} + \Upsilon_{m,t},$$

(43)

with $\rho_m < 1$ and $\Upsilon_{m,t} \sim i.i.d.$

While the the monetary policy instrument in (42) is augmented with the deviation of output and housing prices from trend, for the initial sensitivity analyses it will be parameterised as a pure inflation targeting regime. This is relaxed in section 6, where the model is used to assess the welfare implications of a variety of policy regimes.

This model also features state contingent macroprudential policy via a loan to value rule. Much like a Taylor rule, the regulator adjusts the loan to value ratio ($\chi_t$) from period to period in response to changes in the growth rate of loans. This rule can be written as:

$$\frac{\chi_t}{X} = \left( \frac{b_t}{b_{t-1}} \right)^{-\phi_h}.$$  

(44)

The purpose of this macroprudential policy instrument is to moderate credit booms, and thus mitigate the ability of household indebtedness to amplify adverse shocks. It is a
countercyclical instrument. When times are good, the rule delivers a lower loan to value ratio and limits the further extension of credit to household borrowers. In downturns, the rule permits a higher loan to value ratio and allows the household borrowers to borrow more, and thus alleviate some of the tightening of their budget constraint.

3.6 Equilibrium Conditions

The first order conditions of the household borrowers yield:

\[ C_t^b + q_t \left( H_t^b - (1 - \delta) H_{t-1}^b \right) + r_{t-1}^b \frac{b_{t-1}}{\pi_{c,t}} = b_t + \frac{W_t^b}{P_{c,t}} N_t^b , \]  

(46)

\[ \pi_t^w = \beta^b E_t \pi_{t+1}^w - \lambda^{w, b} \mu_t^b . \]  

(47)

\[ r_t^b \varphi_t = 1 - \beta^b E_t \left\{ \frac{U_{c,t+1}^b}{U_{c,t}^b} \right\} \frac{\pi_t^b}{\pi_{c,t+1}} . \]  

(48)

\[ q_t U_{c,t}^b = U_{h,t}^b + \beta^b (1 - \delta) E_t \left\{ q_{t+1} U_{c,t+1}^b \right\} + (\chi_t) (1 - \delta) q_t U_{c,t}^b \varphi_t \{ \pi_{d,t+1} \} , \]  

(49)

\[ b_t = (\chi_t) (1 - \delta) E_t \left\{ \frac{H_t^b q_{t+1}}{\pi_t^b} \right\} . \]  

(50)

The first order conditions of the household savers yield:

\[ - \mu_t^s U_{n,t}^s = \frac{W_t^s}{P_{c,t}} , \]  

(51)

\[ C_t^s + q_t \left( H_t^s - (1 - \delta) H_{t-1}^s \right) = d_t = \frac{W_t^s}{P_{c,t}} N_t^s + r_{t-1}^d \frac{d_{t-1}}{\pi_{c,t}} + \frac{1}{1 - \omega} \left( \Gamma_{c,t} + q_t \Gamma_{d,t} + (1 - \omega^b) J_t \right) , \]  

(52)

\[ \pi_t^w = \beta^s E_t \pi_{t+1}^w - \lambda^{w, s} \mu_t^s . \]  

(53)

\[ U_{c,t}^s = \beta^s E_t \left\{ U_{c,t+1}^s \right\} \frac{r_{t}^d}{\pi_{c,t+1}} , \]  

(54)

\[ q_t U_{c,t}^s = U_{h,t}^s + \beta^s (1 - \delta) E_t \left\{ q_{t+1} U_{c,t+1}^s \right\} . \]  

(55)

Most of these conditions are standard. (45) and (51) are the household borrowers’ and and savers’ labour supply conditions (inclusive of the time-varying markups \( \mu_t^b \) and \( \mu_t^s \) respectively). (46) and (52) are their budget constraints while (47) and (53) are their New Keynesian Wage Phillips curves. There is an extra term in the Euler equation of the household borrower in (48) as a result of the borrowing constraint, while the Euler
equation of the household savers in (54) is quite conventional. The household borrowers’ demand for housing (given by (49)) also has an extra term, reflecting the additional value an extra unit of housing provides by relaxing the borrowing constraint. (50) is the borrowing constraint.

The financial sector is characterised by:

\[ b_t = d_t + k_t \]  \hspace{1cm} (56)

\[ k_t = (1 - \delta f)k_{t-1} + \omega f j_{t-1} \]  \hspace{1cm} (57)

\[ R_t^b = R_t - \kappa \left( \frac{k_t}{b_t} - \eta \right) \left( \frac{k_t}{b_t} \right)^2 \]  \hspace{1cm} (58)

\[ j_t = r_t^b b_t - r_t^d d_t - \frac{\kappa}{2} \left( \frac{k_t}{b_t} - \eta \right)^2 k_t - \frac{\kappa}{2} \left( \frac{r_m, t}{r^d_{m, t-1}} - 1 \right) r_t^b b_t - \frac{\kappa}{2} \left( \frac{r_d}{r^d_{t-1}} - 1 \right) r_t^d d_t \]  \hspace{1cm} (59)

(56) is the balance sheet identity that all banks must obey. (57) describes how each bank accumulates capital and (58) links the spread between the wholesale banks deposit and loan rates to each bank’s leverage. (59) represents the accumulation of profits by the wholesale and retail branches of the bank in aggregate; part of which are returned to the household saver with the rest retained in order for the bank to accumulate capital.

Finally, assuming flexible loan and deposit rates (ie no cost associated with their change), (60) and (61) link the loan and deposit rates faced by the households in the retail market to the wholesale bank’s loan rate and the policy rate respectively.

\[ r_t^b = \left[ \frac{\epsilon^l}{\epsilon^l - 1} R_t^b \right] \ell_t, \]  \hspace{1cm} (60)

and

\[ r_t^d = \frac{\epsilon^d}{\epsilon^d - 1} R_t. \]  \hspace{1cm} (61)

The optimality conditions from the supply side of the economy are:

\[ Y_{i,j,t} = N_{i,j,t}, \]  \hspace{1cm} (62)

\[ \hat{\pi}_{j,t} = \beta E_t \tilde{\pi}_{j,t+1} + \kappa_j \bar{m} c_{j,t}. \]  \hspace{1cm} (63)

\[ \Gamma_{i,j,t} = P_{i,j,t} Y_{i,j,t} - W_{j,t} N_{i,j,t}, \]  \hspace{1cm} (64)

As discussed previously, (62) and (63) represent the production technology of each firm and the New Keynesian Phillip’s curve for each sector. (64) are the profits in each sector returned to the household savers.

\footnote{The fully specified model, incorporating functional forms is specified in Appendix B.}
Monetary policy, macroprudential policy and aggregate inflation are given by:

\[
\frac{R_t}{R} = \left( \frac{q_t}{q} \right)^{\phi_y} \left( \frac{\pi_t}{\pi} \right)^{\phi_s} \left( \frac{\bar{q}_t}{\bar{q}} \right)^{\phi_h} \epsilon_t, \tag{65}
\]

\[
\frac{\chi_t}{\bar{X}} = \left( \frac{b_t}{b_{t-1}} \right)^{-\phi_h}, \tag{66}
\]

\[
\pi_t = \pi_{c,t}^{1-\alpha} \pi_{h,t}^\alpha, \tag{67}
\]

\[
Y_t = Y_{c,t}^{1-\alpha} Y_{h,t}^\alpha. \tag{68}
\]

4 Parameterisation and Calibration

In the model in this paper, a financial sector intermediates the actions of household borrowers and savers in the markets for loans and savings respectively, while firms undertake the production of non-durables and housing subject to nominal price and wage rigidities. Broadly speaking, the model’s parameter values (detailed in table 1) reflect either consensus in the supporting literature or the careful analysis by the author. In most regards, they can be considered conservative.

Of these parameter values, setting housing’s share of consumption to 20 per cent (\(\alpha = 0.20\)) reflects the fact that housing services have historically averaged 12-13 per cent of GDP; while personal consumption expenditures have, over time, pushed up to over 65 per cent of GDP. While there is little or no evidence that borrowers and savers are represented equally represented in the population, the choice of \(\omega = 0.5\) mirrors the initial choice of Monacelli (2009). The choice of \(\phi = 1\) reflects a compromise between the range of evidence thrown up by the micro- and macroeconometric surveys, while \(\chi = 1\) can be attributed to the fact that both residential investment and housing services have comprised relatively stable components of GDP, despite substantial fluctuations in housing prices over time suggesting, much as is the case with durable goods, unitary elasticity of substitution between housing and non-durable consumption.

The specification of the household borrower and saver discount factors are far from controversial. Setting \(\beta^b\) and \(\beta^s\) to 0.97 and 0.943 respectively is broadly in line with the choices of Iacoviello and Neri (2010). This value of \(\beta^s\) corresponds with a real, annualised deposit rate of just over 2.25 per cent. This represents a markdown of \(e^{\beta^s}\) or 5/6 on the policy rate of 2.75 per cent in steady state. The loan rate is 3.4 per cent, reflecting a markup over the policy rate of \(e^{\beta^b}\) in steady state. The choice of capital adequacy ratio is consistent with Gerali, Neri, Sessa and Signoretti (2010). However, it also broadly corresponds with the requirement of the Basel III regulatory framework that banks hold capital equivalent to 8 per cent of their portfolio, plus a state-dependent capital buffer. Setting \(\delta^I\) to 0.01879 ensures this holds in steady state.

The parameters governing the nominal rigidities are set so that non-durable prices can be expected to hold for 2.5 quarters (\(\theta_c = 0.6\), while house prices are perfectly flexible (\(\theta_h = 0\)). The parameterisation of the nominal wage rigidities follows Carlstrom and Fuerst (2011), who note that if wages are too sticky the responsiveness of housing in the
Table 1: Parameterisation of the Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Share of housing in consumption</td>
<td>0.20</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Proportion of household borrowers in the economy</td>
<td>0.5</td>
</tr>
<tr>
<td>$\beta^b$</td>
<td>Household borrowers discount factor</td>
<td>0.97</td>
</tr>
<tr>
<td>$\beta^s$</td>
<td>Household savers discount factor</td>
<td>0.9943</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Durable depreciation rate</td>
<td>0.01</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Elasticity of Substitution (housing / non-durables)</td>
<td>1</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Inverse of the Frisch Elasticity of Labour Demand</td>
<td>1</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Wholesale bank capital adjustment cost</td>
<td>10</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Capital adequacy requirement</td>
<td>0.09</td>
</tr>
<tr>
<td>$\omega^f$</td>
<td>Proportion of profits retained by the bank each period</td>
<td>0.5</td>
</tr>
<tr>
<td>$\delta^f$</td>
<td>Depreciation of bank capital</td>
<td>0.01879</td>
</tr>
<tr>
<td>$c^l$</td>
<td>Substitutability of retail branches for loans</td>
<td>5</td>
</tr>
<tr>
<td>$c^d$</td>
<td>Substitutability of retail branches for deposits</td>
<td>-5</td>
</tr>
<tr>
<td>$\theta_c$</td>
<td>Probability of not changing non-durable good prices</td>
<td>0.60</td>
</tr>
<tr>
<td>$\theta_h$</td>
<td>Probability of not changing housing prices</td>
<td>0.00</td>
</tr>
<tr>
<td>$\theta_w$</td>
<td>Probability of not changing wages</td>
<td>0.10</td>
</tr>
<tr>
<td>$\phi^w$</td>
<td>Substitutability of household labour types</td>
<td>11</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>Reaction of the monetary instrument to inflation</td>
<td>1.5</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>Reaction of the monetary instrument to the output gap</td>
<td>0</td>
</tr>
<tr>
<td>$\phi_h$</td>
<td>Reaction of the monetary instrument to housing</td>
<td>0</td>
</tr>
<tr>
<td>$\phi_b$</td>
<td>Reaction of the LTV ratio to borrowing</td>
<td>0</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>Monetary policy shock persistence</td>
<td>0.5</td>
</tr>
<tr>
<td>$\rho_f$</td>
<td>Financial shock persistence</td>
<td>0.9</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Technology shock persistence</td>
<td>0.9</td>
</tr>
</tbody>
</table>

The model can greatly exceed what is exhibited by the data. As a result, wages are expected to prevail for slightly longer than a quarter ($\theta_w = 0.10$ - close to the midpoint of the values considered by Carlstrom and Fuerst (2010)) and there is an expected markup on labour of 10 per cent, reflecting relatively substitutable household labour.

5 The Internal Consistency of the Model

This section outlines the properties of the model. It demonstrates that the model conforms with the findings of the numerous studies that have undertaken the structural identification of monetary policy shocks and then uses this context to probe the mechanisms at play in the model and the theory that underpins them. The section then explores how to best use optimal simple rules to manage the volatility wrought on the economy by not only an unanticipated increase in the policy rate, but surprise spike in the loan rate and the marginal cost of production.

5.1 Key Results

The model, under the parameterisation presented in table 1, matches certain characteristics of the data. Primarily, the model exhibits positive co-movement between non-durable
goods and housing in response to a monetary policy shock. The challenges associated with constructing a New Keynesian model that accords with the microevidence on the frequency of price changes and delivers positive co-movement were first documented by Barsky, House and Kimball (2007) and is discussed at length in Perks (2016a).

Furthermore, the response of housing to the monetary policy shock is around 10 times that of non-durable goods - a result that corresponds with the empirical studies of Erceg and Levin (2006) and Cantelmo and Melina (2015).

5.2 The Basic Properties of the Model

A shock to monetary policy filters through the model economy in two ways. It affects the production side of the economy by affecting output in the non-durables sector (by virtue of the nominal rigidities in that sector) while it influences the intertemporal allocation of resources by households as it is directly linked to the rates at which the retail branches

---

8 The choice of a monetary policy shock to tease out the main attributes of the model rests on the fact that it can be interpreted as a policy and demand shock. In addition, it dovetails with a substantial literature on multi-sector models - particularly ones that feature durable goods or housing - and makes comparisons easier.
Figure 2: Properties of the Model - Financial Sector after a Monetary Shock

- Loans
- Deposits
- Retail Loan Rate
- Retail Deposit Rate
- Bank Profits
- Bank Capital
- Loan-to-value ratio
- Tightness of the Borrowing Constraint

Figure 3: Properties of the Model - Production after a Monetary Shock

- Real Wage
- Relative Price of Housing
- Production of Non-Durables
- Production of Housing
- Household Borrower Supply of Labour
- Household Saver Supply of Labour
- Non-durable Good Price Inflation
- Housing Price Inflation
of the banks in the financial sector offer loans and pay deposits. (These effects are summarised in figures 1, 2 and 3.)

As output in the non-durables sector falls, less labour is used in production. Ordinarily, the reductions in the real wage that accompany such a sectoral decline in output would lead to the housing sector snapping up the extra workers at lower per unit rates and expanding even as the non-durable sector contracts. However, the presence of nominal wage rigidities ensures that the overall wage rate adjusts sluggishly and the expansion fails to materialise.

Conversely, the flexible prices in the housing sector mean that the optimal house price is a constant markup over marginal cost. Since labour is the only input into into a linear production function, the wage rate is the marginal cost. The price of durable goods inherits the stickiness in the wage rate and, as a result, the sector also contracts in response to the monetary contraction.

Household borrowers and savers are affected by the shock in a number of ways. Most obviously, the increase in the policy rate changes the cost of transferring consumption through time. The shock to the policy rate drags both the loan and deposit rates up (particularly in a scenario where there is little or no restriction on the intermediary’s adjustment of these rates). Faced with higher repayments in the next period, household borrowers choose to borrow less.

The effect on household savers is more complex. They, in effect, have two vehicles for the transfer of consumption into future periods - either deposits or housing. When the monetary policy shock hits, it lifts the deposit rate and makes saving more attractive. It also drives down the relative price of housing and makes the purchase of housing more appealing. However, rather than increase both, household savers choose to reduce their deposits and increase their holdings of housing. This effect, documented in Perks 2016c, stems from the fact that goods that depreciate over a great many periods have an almost infinite elasticity of intertemporal substitution. The temporary price decline offers an almost unprecedented opportunity to boost their stock of housing - one that they do not hesitate to accept.

This leads to a situation where household savers increase their purchases of durable goods, even as household borrowers are cutting theirs. This can be rationalised by recalling that not only do household savers value future periods more, but they also have greater resources at their disposal from their ownership of profit-making intermediate good producers and financial intermediaries. The net effect is a decline in the purchase of housing - a result largely conditioned by the fall in housing production and the need for markets to clear.

The financial intermediaries also suffer. The reductions in loans and deposits hurt their profits, leading to a fall in bank capital. The twin declines in deposits and bank capital restrict their ability to write loans, although this is partially offset by the countercyclical loan to value ratio.
5.3 Key Mechanisms of the Model

5.3.1 The Contribution of Nominal Wage Rigidities

Nominal wage rigidities are pivotal to the generation of both the positive co-movement between non-durable goods and housing and the relative sensitivity of housing to an unanticipated change in the nominal interest rate. While there are a number of ways to achieve this co-movement, the appeal of nominal wage rigidities stems from the ease with which they can be incorporated and their limited interaction with other variables, ensuring that they don’t markedly alter the model’s dynamics (as can be seen in figure 4).

The effect of nominal wage rigidities is neatly illustrated by allowing the parameter that captures how often wages are able to adjust, \( \theta_w \), to vary across a relatively small range. The chosen range, from 0 to 0.2, reflects the assumption that wages are able to adjust relatively quickly.\(^9\) When wages are perfectly flexible, non-durable and housing purchases move in different directions (and are at odds with the litany of empirical studies that demonstrate that they co-move positively). However, by slowly increasing \( \theta_w \), the positive co-movement properties of the model are greatly enhanced, without affect on the dynamics of other variables. Once \( \theta_w = 0.1 \), there is substantial co-movement and, as noted earlier, housing is around 10 times more sensitive to the monetary shock than non-durable consumption. By the time, \( \theta_w = 0.2 \), this has increased to a factor of almost 30.

This illustrates that small amounts of (and small changes in) wage stickiness can have big effects.

5.3.2 The Contribution of the Borrowing Constraint

The general idea that borrowing constraints can amplify shocks is a well-documented phenomenon. Despite its broad acceptance, this notion hasn’t satisfactorily been explored by multi-sector models that feature durable goods or housing (save the literature based on Iacoviello (2005) which effectively rules out fluctuations in housing production by assuming a fixed supply). Moreover, the experience in the Monacelli (2009) model - the framework that underpins this paper - demonstrates that borrowing constraints in multi-sector models can have the opposite effect. Sterk (2010) shows that, in response to a contractionary monetary policy shock, the presence of a borrowing constraint made the negative co-movement between durable and non-durable consumption more severe, forcing durable consumption to surpass the level that would have been attained without the constraint.

This subsection revisits this by examining the effect of the borrowing constraint in the Monacelli framework with nominal wage rigidities and a financial sector. This can be done by eliminating household borrowers in the economy (or by setting \( \omega = 0 \)). Without them, the borrowing constraint no longer applies, and the overall problem is reduced to that of the household saver. In the initial examination by Monacelli (2009), the absence of

\(^9\)For a survey of the literature on durable goods and housing co-movement, see Perks 2016a.

\(^{10}\)When \( \theta_w = 0 \), they can be changed every period. At the other end of this (limited) spectrum, when \( \theta_w = 0.2 \) wages are able to adjust every 1.25 periods. There is little in the way of a long term restriction on wage variation.
the household borrower curtailed the optimal activities of the household saver by removing their counterparty in the market for savings. But this is not the case in this model. The presence of the financial market, and the independence of the retail loan part of the bank, means that the optimal decision of the household saver is relatively unaffected by their absence. They are still able to make deposits with the financial intermediary.

This has a crucial effect. As discussed previously and seen in figure 4, the household saver responds to the monetary shock by increasing their purchases of housing. As a result, reducing the demand side of the economy to the problem of the household saver effectively ensures that the aggregate response is similar. This is confirmed by figure 5.

In this sense, household borrowers (with their constraints) are far more important in this model compared to the original Monacelli (2009) version. When present in sufficient numbers, they counteract the actions of the household savers and drive both the downturn in housing purchases and the fall in aggregate output in response to the monetary shock. They don’t just amplify the downturn that results from the shock; they generate it.

5.3.3 The Effect of the Financial Sector

Having established that both nominal wage rigidities and the borrowing constraint are required to ensure that the model can match the data (as discussed previously), it remains to be seen how adjustment costs in the financial sector affect the response of the model to shocks.

In the work that introduced this type of financial sector (but that assumed a fixed supply
of housing ala Iacoviello, 2005), Gerali, Neri, Sessa and Signoretti (2010) demonstrated that, while borrowing constraints accelerate or amplify the fall in non-durable consumption that flow from a shock to monetary policy, the presence of adjustment costs in the financial sector attenuate it. These adjustment costs slow the adjustment of the retail deposit and loan rates (and, more broadly, the economy) to changes in the policy rate. But their effect in a model where borrowers and their constraint make a more fundamental contribution by, in effect, driving the fall in output, remains to be seen.

These adjustment costs are captured by the parameters $\kappa_l$ and $\kappa_d$ and the adjustment processes are given by [21] and [26] respectively. When these parameters are zero, the retail branches of the financial intermediaries can adjust their rates costlessly. If we assume the costs associated with adjusting these rates are symmetric, increasing $\kappa_l$ and $\kappa_d$ from zero towards the range described in Gerali, Neri, Sessa and Signoretti (2010) should give some indication of the effect of these adjustment costs given the multi-sector nature of the model.

The findings, while contrasting with those of Gerali, Neri, Sessa, Signoretti (2010), are instructive. Even at levels well below the range of 10 to 15 utilised by Gerali, Neri, Sessa, Signoretti (2010), the adjustment costs quickly dampen the reaction of the loan and deposit rate to the shock. The loan and deposit rate effectively resist the unanticipated change in monetary policy and do not act as a transmission mechanism for the shock. Even when the adjustment costs are present at low levels, in the context of a monetary policy shock they have minimal impact on the other variables.
5.3.4 The Impact of Policy Settings

The introduction of a loan to value ratio and bank capital requirements into the model also affect the response of the model to shocks. While the question of optimal policy - the way in which these instruments *should* operate so as to stabilise the economy - is addressed in the subsequent section, as a prelude this subsection examines the effects of changes in the levels of these instruments.

Any increase in the steady state loan to value ratio ($\chi_t$) means that a larger amount of the overall stock of housing held by the household borrower can be pledged as collateral. All else being constant, a higher $\chi_t$ relaxes the borrowing constraint. In this model, $\chi_t$ is state dependent and set by a Taylor-type rule. But were it set at a constant level in every period (as is the case in many models), changes in this variable would have substantial effects.

To examine this, $\chi_t$ is allowed to take an number of values from 0 to 1. The former of these ($\chi_t = 0$) implies no borrowing, as no proportion of the housing stock can be used as collateral. (There are still household borrowers in the economy, however they are severely constrained.) The latter ($\chi_t = 1$) ensures that, so long as the expected price in the next period of the undepreciated housing stock currently owned by the borrower covers the interest owed on today’s borrowings, the household borrower is able to borrow an additional unit. (This effectively implies costless monitoring and repossession of collateral in the event of a default.)

As the constraint on the household borrower is relaxed (or as $\chi$ increases), the households
Figure 7: The Impact of the Loan to Value Ratio

have differing reactions to the monetary shock. Purchases by the household borrower fall, while those of the household saver increase. The explanation for this is simple. The spike in the nominal interest rate means higher loan rates for household borrowers. Since higher levels of $\chi$ imply more outstanding loans in steady state that need to be rolled over, it also ensures that this higher loan rate is applied to higher levels of borrowing, further compounding the tightening of the borrower’s budget constraint. On the other hand, household savers are able to earn a better return on their savings, and thus experience a relaxing of their budget constraint (figure 7).

When a financial intermediary is subject to an increase in their bank capital requirements ($\eta$), they are forced to hold capital per loan initiated (easily seen from (12)). As such, it makes it more costly to lend. In this model, the bank capital requirements are exogenous. Yet because they imply a restriction on borrowing, they have substantial effects.

To illustrate this, $\eta$ is allowed to take an number of values from 0.07 to 0.11. This spans the range of values thrown up by the succession of Basel Accords. (Initially, $\eta$ was 8 percent in Basel I, but has since increased to incorporate a buffer in Basel III).

As the bank capital requirement is increased (or as $\eta$ increases), capital held by the bank is increased (to the detriment of profits) in steady state. However, this has a minimal effect on the responsiveness of the model to the monetary policy shock, leading to the conclusion that, assuming this model is an accurate reflection of the macroeconomy, and unanticipated changes in monetary policy are the policymakers major source of concern, bank capital requirements are a relatively ineffective tool for mitigating their effects (figure 8).
6 Welfare Analysis

Having put together a model that utilises nominal wage and price rigidities and a borrowing constraint to match the key characteristics of the data, there is an open question as to what this model can tell us about policy. Specifically, whether there is any value in an active macroprudential policy that targets financial aggregates such as the loan to value ratio or the amount of loans issued by banks when monetary, financial and supply shocks are hitting the economy.

To this end, this section numerically evaluates and contrasts the societal welfare that results from a series of plausible and implementable policy rules.

There are two common approaches to welfare analysis in DSGE models. In the first, in lieu of a policy rule that closes the model, a benevolent and omniscient social planner maximises their own objective function - usually similar to the utility maximisation problem of the households - subject to the optimality conditions offered by the model. This reflects the optimal policy under commitment (also called Ramsey policy). The second involves solving the model using a second-order approximation to the structural equations and evaluating welfare using this solution. In the first approach, implementable policy rules are ranked and contrasted with the optimal (but not implementable) policy. In the second, the rules are ranked, but without reference to the welfare measure associated with the Ramsey allocation.

This paper follows Rubio and Carrasco-Gallego (2014) in adopting the latter approach. As such, it focuses on the welfare changes that result from a selection of implementable
rules and abstracting from Ramsey policy.

To this end, the welfare of the household borrower and the household saver are defined as:

\[ V_0^b = E_0 \left\{ \sum_{t=0}^{\infty} \beta_t^b \left[ \log(X_t^b) - \frac{N_t^{b(1+\phi)}}{1 + \phi} \right] \right\}, \quad \text{(69)} \]

\[ V_0^s = E_0 \left\{ \sum_{t=0}^{\infty} \beta_t^s \left[ \log(X_t^s) - \frac{N_t^{s(1+\phi)}}{1 + \phi} \right] \right\}, \quad \text{(70)} \]

where \( X_t^k \) and \( N_t^k \) are contingent plans for consumption (comprising non-durable consumption and housing) and labour (\( k \in \{b, s\} \)).

The welfare of each type of household is weighted not only by the proportion of the population they represent, but also by their discount factor to ensure that they receive similar amounts of utility from a constant stream of consumption. This yields an aggregate measure of societal welfare given by:

\[ V_t = \omega(1 - \beta_t^b)V_t^b + (1 - \omega)(1 - \beta_t^s)V_t^s. \quad \text{(71)} \]

The rather abstract notion of welfare can be rendered more concrete by representing the difference in welfare in terms of consumption. The notion can be simply expressed as the constant proportion of consumption a household would be willing to forgo in order to move to a better policy rule, or one that offers them greater welfare. Since it cannot be assumed that the households will respond symmetrically, or even similarly, to the introduction of a new policy rule, expressing consumption equivalents in this way also permits the teasing out of the winners and losers of a particular change in policy.

Consumption equivalents for the two types of households are given by:

\[ CE_b = exp \left[ (1 - \beta^b) \left( X_b - V_t^{b,s} \right) \right] - 1, \quad \text{(72)} \]

\[ CE_s = exp \left[ (1 - \beta^s) \left( V_s - V_t^{s,b} \right) \right] - 1, \quad \text{(73)} \]

where \( V_t^{k,s} (k \in \{b, s\}) \) represents the household welfare associated with the benchmark policy rule; the traditional inflation targeting regime (where \( \phi_\pi = 1.5 \)). A positive value for \( CE^k \) suggests that the alternative policy rule is superior and offers an improvement in welfare. A negative value reflects that the alternative rule offers less welfare and the household would have to be offered additional consumption to accept it.

### 6.1 Welfare Comparison of Policy Rules

The policy experiment in this paper contrasts the effect on welfare of several different policy rules in the wake of three different exogenous shocks. They are:

- an inflation targeting regime - the policy rate responds only to inflation. Moreover, the parameter governing the response of the interest rate to inflation is positive and greater than 1 so as to punish any deviation from trend inflation (\( \phi_\pi = 1.5 \)).
• a simple Taylor Rule - a positive output gap results in a higher policy rate ($\phi_x = 1.5$, $\phi_y = 0.5$)
• a Taylor Rule augmented with house prices - the policy rate leans against increases in housing prices ($\phi_x = 1.5$, $\phi_y = 0.5$, $\phi_h = 1.0$)
• a simple Taylor Rule with an endogenous loan to value ratio ($\phi_x = 1.5$, $\phi_y = 0.5$, $\phi_b = -1.0$), and
• an augmented Taylor Rule with an endogenous loan to value ratio ($\phi_x = 1.5$, $\phi_y = 0.5, \phi_h = 1.0, \phi_b = -1.0$).

Each of these can be viewed as special cases (or combinations) of (42) and (44).

As described in section 3, the monetary shock is an unanticipated increase in the policy rate, the financial shock is a surprise in the loan rate only and the productivity (or technology) shock is an improvement in the intermediate producers’ ability to transform labour into intermediate goods.

<table>
<thead>
<tr>
<th>Policy Rule</th>
<th>Monetary Shock</th>
<th>Productivity Shock</th>
<th>Financial Shock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V_t$</td>
<td>$CE^b$</td>
<td>$CE^c$</td>
</tr>
<tr>
<td>Inflation targeting              -86.9484</td>
<td>0</td>
<td>0</td>
<td>-86.7638</td>
</tr>
<tr>
<td>Simple Taylor Rule (STR)         -86.8328</td>
<td>-0.00103</td>
<td>0.00151</td>
<td>-82.694</td>
</tr>
<tr>
<td>Augmented Taylor Rule (ATR)      -86.7316</td>
<td>-0.00186</td>
<td>0.00260</td>
<td>-83.1892</td>
</tr>
<tr>
<td>STR with Endogenous LTV ratio    -86.8284</td>
<td>-0.00105</td>
<td>0.00156</td>
<td>-82.3683</td>
</tr>
<tr>
<td>ATR with Endogenous LTV ratio    -86.7521</td>
<td>-0.00187</td>
<td>0.00260</td>
<td>-82.5731</td>
</tr>
</tbody>
</table>

While they do not point to a single policy rule as the best insulation against the shocks considered in this paper, the results in table 2 do offer some interesting takeaways. Without optimising across parameter combinations, these results cannot be considered conclusive, but rather indicative.

In the wake of a monetary policy shock, social welfare is highest under an Taylor rule that targets inflation, the output gap and house prices. Similar levels of welfare are generated with or without macroprudential policy (via the endogenous loan to value rule). The story is different after a positive productivity shock. A Taylor Rule that targets only inflation and the output gap (and not house prices) outperforms the others (and the inflation targeting regime by a substantial amount). And none of the rules distinguished themselves after an unanticipated increase in the borrowing rate.

More stark are the welfare tradeoffs implied by each rule. While each of the rules examined offer greater social welfare than under a simple inflation targeting regime, they overwhelmingly favour household borrowers. Positive $CE^b$ imply that these rules offer savers greater consumption, while the opposite is true for borrowers (with negative $CE^b$). This tradeoff becomes more acute as the parameter values in the policy rule increase (particularly $\phi_b$). The reason for this is that, as opposed to a number of other models where the savior lends directly to the borrower, the financial intermediaries here allow the agents to pursue their objectives without the restriction of trying to find a counterparty in the market for savings. Borrowing and savings do not have to equal each other. Rules that are designed to increase the policy rate in response to adverse changes in macroeconomic
variables, or reduce the amount that can be borrowed in any given period are to the detriment of the household borrowers, who rely on them to relax their budget constraint. The household savers on the other hand benefit from a higher deposit rate (linked to the policy rate), or the fall in house prices that follows of the household borrower’s inability to borrow to buy housing. In this sense, the finding dovetails nicely with the work of Rubio and Carrasco-Gallego (2015).

7 Conclusion

This paper develops a New Keynesian model with fully developed non-durable production and housing sectors, a financial sector and macroprudential policy in the form of bank capital requirements and a loan to value ratio. The internal dynamics of the model are teased out through its response to an adverse monetary policy shock. The model exhibits positive co-movement between non-durable consumption and housing, ensuring that not only does the model line up with a number of empirical studies but that it is not subject to one of the pitfalls of multi-sector models (featuring at least one sector with flexible prices).

On the policy front, the model explores a number of monetary and macroprudential policy instruments. It finds that a state contingent loan to value ratio, or even monetary policy that expressly targets deviations in house prices from trend are, from the perspective of social welfare, the most viable in the wake of an unanticipated increase in the policy rate. This is not the case after a technology shock, where simple Taylor Rules are superior. None of the rules substantially outperform the inflation targeting regime after a surprise hike in the borrowing rate. Absent a consensus (and leaving aside all questions of communication and the ease of transition from one rule to another), policymakers may be best served by conditioning their choice of rule on their assessment of the shocks hitting the economy, rather than trying to set in place the one rule that supplants all others.

While all of these rules represent an improvement over the inflation targeting regime, the improvement in welfare comes with a tradeoff. The gains in social welfare predominately result from increases in the consumption equivalents of household savers, while household borrowers find themselves worse off.

This model has considerable scope for policy use in the future. It is a plausible model environment for the identification of optimal policy and its exploration of macroprudential policy could be expanded via endogenous capital adequacy requirements or via the inclusion of endogenous default (as in Forlati and Lambertini 2011) or more specific housing sector analysis. In these and many other regards, this model appears a good (and relatively complete) foundation for future research projects.
References


[34] Perks, C., 2016a. A note on production linkages and the durable good co-movement puzzle, mimeo.


Appendix A

Table 3: Comparable Models and their Characteristics

<table>
<thead>
<tr>
<th>Author</th>
<th>Housing Production</th>
<th>Financial Sector</th>
<th>Macropolicy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Davis and Heathcote (2005)</td>
<td>✓</td>
<td>×</td>
<td>×</td>
</tr>
<tr>
<td>Daraq Paries et al (2008)</td>
<td>✓</td>
<td>×</td>
<td>Loan to value ratio</td>
</tr>
<tr>
<td>Gerali et al (2009)</td>
<td>×</td>
<td>✓</td>
<td>Loan to value ratio</td>
</tr>
<tr>
<td>Iacoviello and Neri (2010)</td>
<td>✓</td>
<td>×</td>
<td>Loan to value ratio</td>
</tr>
<tr>
<td>Lambertini et al (2013)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Mendicino (2012)</td>
<td>✓</td>
<td>×</td>
<td>Loan to value ratio</td>
</tr>
<tr>
<td>Falagiarda and Saia (2013)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
<tr>
<td>Justiniano et al (2015)</td>
<td>×</td>
<td>✓</td>
<td>Loan to value ratio</td>
</tr>
<tr>
<td>Justiniano et al (2015)</td>
<td>✓</td>
<td>×</td>
<td>Loan to value ratio</td>
</tr>
<tr>
<td>Perks (2016)</td>
<td>✓</td>
<td>✓</td>
<td>✓</td>
</tr>
</tbody>
</table>

Appendix B

7.1 Equilibrium Conditions

A competitive equilibrium is defined by functions for \((C_t^b, C_t^s, C_t, H_t^b, H_t^s, H_t, N_t^b, N_t^s, N_{c,t}, N_{h,t}, Y_{c,t}, Y_{h,t}, Y_t, F_t, w_t, q_t, \pi_t^b, \pi_t^s, \pi_t^c, \pi_t, \chi_t, mc_{c,t}, mch_{h,t}, \mu_t^w, \mu_t^w, \mu_t^b, dt_t, kt_t, j_t, r_t, R_t, H_t^b, \Gamma_{c,t,G_{h,t}, \varphi_t, \varepsilon_{m,t}, \varepsilon_{l,t}, \varepsilon_{a,t}})\) satisfying the following system of equations:

\[
\mu_t^{w,b} N_t^b C_t^b /(1 - \alpha) = w_t, \tag{74}
\]

\[
q_t (1 - \alpha)/C_t^b = \alpha/H_t^b + \beta^b q_{t+1} (1 - \delta)(1 - \alpha)/C_{t+1}^b + \chi_t (1 - \delta)q_{t+1} \pi_t^c \pi_{t+1}^c (1 - \alpha)/C_t^b, \tag{75}
\]

\[
r_t^b \varphi_t = 1 - \beta^b \mathbb{E}_t \left\{ \frac{C_t^b}{C_{t+1}^b} \frac{r_t^b}{\pi_{c,t+1}} \right\}, \tag{76}
\]

\[
C_t^b + q_t \left( H_t^b (1 - \delta) H_{t-1}^b \right) + r_{t-1}^b \frac{b_{t-1}}{\pi_{c,t}} = b_t + w_t N_t^b, \tag{77}
\]

\[
b_t = \left( \chi_t \right) (1 - \delta) \mathbb{E}_t \left\{ \frac{H_t^b q_{t+1}}{r_t^b} \frac{1}{\pi_{c,t+1}} \right\}, \tag{78}
\]

\[
\hat{n}_t^{w,b} = \beta^s \mathbb{E}_t \hat{n}_{t+1}^{w,b} - \lambda^{w,b} \mu_t^{w,b}, \tag{79}
\]

\[
\mu_t^{w,s} N_t^s \pi_t^s C_t^s /(1 - \alpha) = w_t, \tag{80}
\]

\[
q_t (1 - \alpha)/C_t^s = \alpha/H_t^s + \beta^s q_{t+1} (1 - \delta)(1 - \alpha)/C_{t+1}^s, \tag{81}
\]
\[ 1 = \beta^s E_t \left\{ \frac{C_t^s}{C_{t+1}^s} \frac{r_t^d}{\pi_{c,t+1}} \right\} \left( \frac{C_t^s + q_t (H_t^s - (1 - \delta) H_{t-1}^s)}{1 - \omega} \right) \]

\[ C_t^s + q_t \left( H_t^s - (1 - \delta) H_{t-1}^s \right) \left( d_t = w_t N_t^s + r_{t-1}^d d_{t-1} + \frac{1}{1 - \omega} \left( \Gamma_{c,t} + q_t \Gamma_{d,t} + (1 - \omega^b) \pi_t \right) \right), \]

\[ \beta_t^w = \beta^s E_t \beta_{t+1}^w - \lambda^w \beta_t^w, \]

\[ b_t = d_t + k_t, \]

\[ k_t = (1 - \delta^f) k_{t-1} + \omega^f j_{t-1}, \]

\[ R_t^b = R_t - \kappa \left( \frac{k_t}{k_t - \eta} \right) \left( \frac{k_t}{k_t} \right)^2, \]

\[ j_t = r_t^b b_t - r_t^d d_t - \frac{\kappa}{2} \left( \frac{k_t}{k_t - \eta} \right)^2 k_t - \frac{\kappa}{2} \left( \frac{r_m^t}{r_m^t - 1} \right) r_t^b b_t - \frac{\kappa}{2} \left( \frac{r_t^d}{r_t^d - 1} \right) r_t^d d_t, \]

\[ r_t^b = \left[ \frac{e^t}{e^t - 1} R_t^b \right] \left[ e^t \right] \]

\[ r_t^d = \left[ \frac{e^d}{e^d - 1} R_t^d \right] \]

\[ Y_{c,t} = A_t N_{c,t}, \]

\[ \pi_{c,t} = \beta E_t \pi_{c,t+1} + \kappa_c \pi_{c,t}, \]

\[ m_{c,t} = w_t N_{c,t} / Y_{c,t}, \]

\[ \Gamma_{c,t} = Y_{c,t} - w_t N_{c,t}, \]

\[ Y_{h,t} = A_t N_{h,t}, \]

\[ \pi_{h,t} = \beta E_t \pi_{h,t+1} + \kappa_h \pi_{h,t}, \]

\[ m_{c,t} = w_t N_{c,t} / q_t Y_{h,t}, \]

\[ q_t \Gamma_{h,t} = q_t Y_{h,t} - w_t N_{h,t}, \]

\[ C_t = \omega C_t^b + (1 - \omega) C_t^s, \]
\[ H_t = \omega H_t^b + (1 - \omega) H_t^s, \]  
\[ N_{h,t} = \omega N_t^b + (1 - \omega) N_t^s - N_{c,t}, \]  
\[ Y_{h,t} = \omega (H_t^b - (1 - \delta) H_{t-1}^b) + (1 - \omega) (H_t^s - (1 - \delta) H_{t-1}^s), \]  
\[ F_t = H_t - (1 - \delta) H_{t-1}, \]  
\[ \pi_{c,t} = \pi_{h,t} q_{t-1} / q_t, \]  
\[ \pi_{w,b}^t = \pi_{c,t} w_t / w_{t-1}, \]  
\[ \pi_{w,b} = \pi_{w,s}, \]  
\[ \frac{R_t}{R} = \left( \frac{\pi_t}{\bar{\pi}} \right)^{\phi_x} \varepsilon_t, \]  
\[ \frac{\chi_t}{\bar{\chi}} = \left( \frac{\theta_t}{\theta_{t-1}} \right)^{-\phi_h}, \]  
\[ \pi_t = \pi_{c,t}^{1-\alpha} \pi_{h,t}^\alpha, \]  
\[ Y_t = Y_{c,t}^{1-\alpha} Y_{h,t}^\alpha, \]  
\[ log\varepsilon_{m,t} = \rho_m log\varepsilon_{m,t-1} + Y_{m,t}, \]  
\[ log\varepsilon_{l,t} = \rho l log\varepsilon_{l,t-1} + Y_{l,t}, \]  
\[ log\varepsilon_{a,t} = \rho a log\varepsilon_{a,t-1} + Y_{a,t}. \]