# Is the RBA Averse to Interest Rate Volatilty?

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**Abstract**

Monetary policy decisions by the Reserve Bank of Australia are often scrutinised in the media as to whether the rate adjustment is too frequent, or too little. More often than not, central banks practice inertial behaviour in interest rate settings, referred to as interest rate smoothing. This paper benchmarks historical analysis of RBA policy against optimally derived interest rate rules, and draws on control methods (Ball, 1999a and Rudebusch and Svensson, 1999) to determine an optimal Taylor rule incorporating interest rate smoothing. Smoothing reduces volatility in the real interest rate, lending credence to the fact that smoothing can help reduce uncertainty in financial markets. However, the tradeoff of smoothing is that it dampens the response of optimal monetary policy to inflation and output, which take longer horizons to stabilise. Comparisons with optimal rules suggest that the Reserve Bank is averse to interest rate volatility, with a recommendation that they change interest rates more frequently and with greater magnitude in response to inflation and output deviations from their equilibrium values.

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# Contents

# 1 Introduction

There is considerable debate as to whether Reserve Bank of Australia (RBA) cash rate decisions should verge on the side of caution or whether rates are moved too aggressively[1]. In practice, interest rate settings are often inertial, with gradual adjustment of interest rates by central banks. Interest rate smoothing is featured in central banking for a number of reasons. For example, central banks may want to keep interest rates at a given level to avoid disruption in financial markets (Goodfriend, 1991), and because of the uncertainty with respect to the basis for an interest rate change, viz. the current state of the economy (Sack, 1998).

A common strand of literature (Clarida et al, 1998 and De Brouwer and Gilbert, 2005) incorporate the smoothing process in monetary policy via a Taylor rule, a simple monetary rule which prescribes how a central bank should adjust its interest rate policy instrument in a systematic manner in response to developments in inflation and macroeconomic activity. Much of the work in characterising central bank reaction functions is purely empirical, and does not answer the more pertinent question of whether the central bank in fact practices optimal monetary policy. This paper extends on historical analysis of RBA policy by comparing policy rates with optimally derived interest rate rules.

To shed light on the optimal rate of smoothing by the central bank, the paper draws on control theory methods (Ball, 1999a and Rudebusch and Svensson, 1999) to determine optimal interest rate rules that incorporate smoothing behaviour. An optimal control problem is solved in which the Taylor rule is a linear feedback rule that captures a relationship between the interest rate, the control variable, and state variables such as inflation and the output gap. Optimal monetary policy is then defined as setting the parameters in a Taylor rule to minimise a loss function[2] subject to the constraints of an economic model.

A key finding of the paper is that the optimal coefficients in a Taylor rule are dependent on the degree of preference for smoothing in the loss function. An advantage of smoothing is less volatility in the real interest rate, lending credence to the fact that interest rate smoothing can help reduce uncertainty in financial markets. However, the cost of smoothing is longer time horizons for inflation and output

[1] In the Australian Financial Review, articles suggest alternative views of whether the central bank cuts rates too often, or without reason. A letter to the editor dated 10th of May, 2013, says that rates should not change more than once a quarter and should be meaningful. Mardi Dungey in an editorial column argues the contrary view, that the unpredecented rate cut to 2.75 per cent in May is not required.

[2] A central bank loss function is typically composed of both inflation and output variance.

to stabilise.

In merging the theoretical control approach with comparisons of historical policy, monetary policy by the RBA is characterised as an interest rate rule via methods in Clarida et al (1998). Comparisons with optimal rules suggest that the rate of interest rate smoothing by the RBA is excessive, with a recommendation that they move rates more frequently and with greater magnitude in response to inflation and output deviations from their equilibrium values.

The paper is outlined as follows. Section 2 provides a basic introduction to the Taylor rule. In section 3, optimal monetary policy is derived for the case when there is zero smoothing. In section 4, optimal policy incorporating interest rate smoothing is developed. In section 5, Taylor rules are estimated to capture empirical inflation and output gap coefficients for Australia, and RBA policy settings are then compared to optimal interest rate rules. Section 6 concludes the dissertation, and section 7 contains appendices and references.

# 2 Background

## 2.1 Taylor rule

The Taylor rule provides a useful framework for the analysis of historical monetary policy and for the evaluation of policy decisions. By committing to follow a rule, policymakers can avoid the inefficiency associated with the time-inconsistency problem (Kydland and Prescott, 1977 and Barro and Gordon, 1983). A Taylor rule also enhances the transparency and accountability of the central bank, and enables forecasting by financial market participants, businesses and households.

Seminal work by Taylor (1993) calibrated a simple interest rate rule for the US economy, where the interest rate is dependent on deviations of inflation and output from their target levels. Let $r_n$ be the natural rate of interest, $(\pi_t - \pi^T)$ be the gap between the annual rate of inflation and a target rate, and $(y_t - y_n)$ the output gap [3]. Taylor sets the interest rate as follows,

$$i_t = r_n + \pi_t + (\theta_\pi - 1)(\pi_t - \pi^T) + \theta_y(y_t - y_n) \tag{2.1}$$

Taylor assumes the natural rate, inflation target and trend growth are all equal to 2 per cent for the US economy, and the inflation and output gap parameters, $\theta_\pi$ and $\theta_y$, are equal to 1.5 and 0.5

[3] All these variables are expressed in per cent per annum

respectively[4]. Thus, equation (2.1) takes the more specific form, $i_t = 2 + \pi_t + 0.5(\pi_t - 2) + 0.5(y_t - 2)$.

Taylor finds the empirical rule closely follows the Fed funds rate in the 1987 to 1992 period. The Taylor rule *leans against the wind*, by keeping interest rates high when inflation is above target or output is above potential, and vice versa (Bernanke, 2004 and Carlstrom and Fuerst, 2003). A consensus exists that the rule should not be followed rigidly, but rather be used as a guidepost in setting monetary policy[5].

An important aspect of the rule is the Taylor principle, a condition in which the interest rate responds to inflation by more than a one-for-one basis. This condition characterises monetary policy as being an inflation fighter, and explains why many countries experienced low and stable levels of inflation since the 1990s, in a period known as the *great moderation* (Taylor, 1998).

### 2.2 Interest rate smoothing

In practice, a central bank is often inertial in setting interest rates. The basic Taylor rule, when supplemented by the addition of the lagged nominal interest rate, does quite well in matching actual behaviour. Rules with interest rate smoothing are used widely in the literature (Clarida et al, 1998, Orphanides, 2001 and Lee et al, 2011). Including the lagged interest rate in a Taylor rule can account for serial correlation in estimation. The parameter $\rho$ captures the degree of smoothing, with a higher $\rho$ indicating more inertia in setting interest rates.

$$i_t = \rho i_{t-1} + (1 - \rho)\left[r_n + \pi_t + (\theta_\pi - 1)(\pi_t - \pi^T) + \theta_y(y_t - y_n)\right] \quad 0 < \rho < 1 \tag{2.2}$$

The nominal interest rate is a weighted average of the lagged interest rate and the original Taylor rule, with weights $\rho$ and $1 - \rho$, respectively.

## 3 Optimal Monetary Policy

### 3.1 Central bank utility function

To define optimality requires an evaluation of a rule's performance, and this is often measured by a central bank welfare loss function. The current literature (Woodford, 2001, Gali, 2008 and Blanchard

[4]There is no theoretical justification that these parameters are optimal, and have been chosen in a rather ad hoc manner.

[5]The Taylor rule has a number of limitations. For example, it rests on the assumption of a constant natural rate of interest, $r_n$, which may vary over time in response to any disturbances to technology or the marginal product of capital (Woodford, 2001). The output gap is dependent on an unobservable variable, the natural evel of output $y_n$, and output gap estimates may be uncorrelated with shocks to $y_n$.

and Gali, 2008) derive a central bank loss function as an approximation to the expected discounted utility of a household, which is equal to the sum of quadratic changes in inflation and the output gap $\widetilde{y}_t$, where $V_t$ is the period utility and $\beta$ is the discount factor.

$$E_t \sum \beta^i V_{t+i} \approx -\Omega E_t \sum \beta^i \left[\pi_{t+i}^2 + \lambda \widetilde{y}_{t+i}^2\right] \tag{3.1}$$

Central banks, such as the US Federal Reserve, pursue a dual mandate[6], requiring monetary policy to maintain price stability and full employment concurrently. Minimising inflation variance is consistent with central banking's emphasis that price stability is the long-run, overriding goal of monetary policy (Mishkin, 2007), and the output variance term captures the full employment objective. Inflation targeting is a framework that minimises a quadratic loss function of the form,

$$L = \sum_{t=0}^{\infty} \beta^t \left[(\pi_t - \pi^T)^2 + \lambda_y \widetilde{y}_t^2\right] \tag{3.2}$$

where $\pi^T$ is the inflation target, and $\beta$ is the discount rate[7] (Walsh, 2011). The central bank can choose to pursue a strict inflation target, by setting $\lambda_y = 0$, or to practice a flexible inflation target, in which monetary policy also responds to output deviations. A central bank which practices interest rate smoothing will target a loss function that also controls for interest rate deviations[8] (Clarida et al, 1998 and Orphanides, 2001).

$$L = \sum_{t=0}^{\infty} \beta^t [(\pi_t - \pi^T)^2 + \lambda_y \widetilde{y}_t^2 + \lambda_i (i_t - i_{t-1})^2] \tag{3.3}$$

## 3.2 Model

The model consists of four equations (Ball, 1999a). The first two equations are the Phillips and IS curves respectively, and capture the behaviour of inflation, $\pi_t$ and the output gap, $y_t - y_n$, denoted $\widetilde{y_t}$.

$$\pi_t = \pi_{t-1} + \phi \widetilde{y_{t-1}} \quad , \phi > 0 \tag{3.4}$$

The IS curve posits a negative relationship between the output gap and the lagged real rate $r_{t-1}$. The error $u_t$ captures any shock to the natural level of output,

$$\widetilde{y_t} = -\alpha r_{t-1} + u_t \quad , \alpha > 0 \tag{3.5}$$

The second set of two equations capture the Fisher equation, in which the real rate is equal to the nominal rate less expected inflation, and inflation expectations. For simplicity in this model, adaptive expectations are assumed, in which the future expectation of inflation is equal to its current value.

[6]Based on the 1977 Federal Reserve Act, which states their mandate is to promote effectively the goals of maximum employment, stable prices, and moderate long-term interest rates. See Mishkin (2007)

[7]If $\beta = 1$, then there is no discounting of future losses in inflation and output. If $\beta < 1$, then future periods of inflation and output are weighted less, implying central banking becomes more myopic in their goal-setting.

[8]Credit and labour market frictions account for other distortions that monetary policy should attempt to minimise (Blanchard and Gali, 2008 and Walsh, 2011).

$$r_t = i_t - E_t\pi_{t+1} \quad , E_t\pi_{t+1} = \pi_t \tag{3.6}$$

The lagged real rate in the IS curve then becomes the difference $i_{t-1} - \pi_{t-1}$.

$$\tilde{y_t} = -\alpha(i_{t-1} - \pi_{t-1}) + u_t \tag{3.7}$$

## 3.3 Analytical solution

A derivation of the interest rate rule, based on Ball (1999a) is provided in the appendix.The optimal rule is a function of the IS and Phillips curve parameters, $\alpha$ and $\phi$, and inflation preferences $\lambda$.

$$i_t = \left(1 + \frac{q^*}{\alpha}\right)\pi_t + \frac{\phi q^*}{\alpha}\tilde{y_t}, \quad q^* = \frac{-\phi\lambda + \sqrt{\phi^2\lambda^2 + 4\lambda}}{2} \quad , \quad \theta_\pi = 1 + \frac{q^*}{\alpha} \ , \theta_y = \frac{\phi q^*}{\alpha} \tag{3.8}$$

There are some important implications from this analysis. Firstly, the optimal rule satisfies the Taylor principle, in which the nominal interest rate responds to inflation by more than a one-for-one basis, $\theta_\pi > 1$.

When $\lambda$ approaches infinity, the central bank pursues strict inflation targeting, and it can also be shown that $q^*$ approaches $\frac{1}{\phi}$, with $\theta_\pi \to 1 + \frac{1}{\alpha\phi}$ and $\theta_y \to \frac{1}{\alpha}$. In contrast to output targeting, inflation targeting is a more aggressive policy for setting interest rates. Table 1 summarises the interest rate rules for each policy.

Table 1: Interest rate rules for inflation and output targeting monetary policies

| Policy | Loss function | $\theta_\pi$ | $\theta_y$ |
|---|---|---|---|
| Flexible inflation targeting | $L = \sum_{t=0}^{\infty}[\lambda\pi_t^2 + \tilde{y}_t^2]$ | $1 + \frac{q^*}{\alpha}$ | $\frac{\phi q^*}{\alpha}$ |
| Strict inflation targeting | $L = \sum_{t=0}^{\infty}\pi_t^2$ | $1 + \frac{1}{\alpha\phi}$ | $\frac{1}{\alpha}$ |
| Strict output targeting | $L = \sum_{t=0}^{\infty}\tilde{y_t}^2$ | 1 | 0 |

### 3.3.1 Calibration of model

A numerical analysis will use a set of base parameter values from Ball (1999a). The IS coefficient $\alpha$ is one, which is to say a one per cent rise in the interest rate will cause the output gap to fall by one per cent. The Phillips curve parameter $\phi$ is equal to 0.4. The relative importance of inflation in the loss function, $\lambda$, is equal to 2, and is consistent with RBA preferences as a flexible inflation targeter (Otto and Voss, 2011).

### 3.3.2 Determinacy region

Monetary policy determinacy is defined as the conditions required on monetary rules in order for a stable economic equilibrium to exist[9]. Converting the equations (3.4) and (3.7) into state space form,

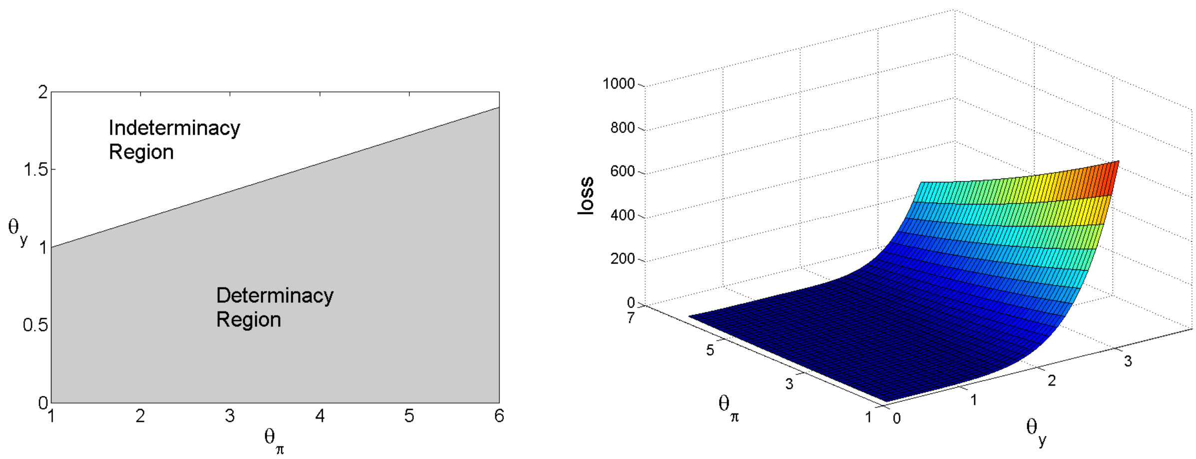
$$\begin{bmatrix} \pi_{t+1} \\ \widetilde{y_{t+1}} \end{bmatrix} = \begin{bmatrix} 1 & \phi \\ \alpha & 0 \end{bmatrix} \begin{bmatrix} \pi_t \\ \widetilde{y_t} \end{bmatrix} + \begin{bmatrix} 0 \\ -\alpha \end{bmatrix} i_t \tag{3.9}$$

and substituting the interest rate rule, $i_t = \begin{bmatrix} \theta_\pi & \theta_y \end{bmatrix} \begin{bmatrix} \pi_t \\ \widetilde{y_t} \end{bmatrix}$ into equation (3.9), we obtain,

$$\begin{bmatrix} \pi_{t+1} \\ \widetilde{y_{t+1}} \end{bmatrix} = \begin{bmatrix} 1 & \phi \\ \alpha(1-\theta_\pi) & -\alpha\theta_y \end{bmatrix} \begin{bmatrix} \pi_t \\ \widetilde{y_t} \end{bmatrix} \tag{3.10}$$

Computation of eigenvalues is based on the matrix in equation (3.10). The region is shown for the calibrated parameters in Figure 1. A three dimensional view with the loss on the Z axis is an alternative way to illustrate the determinacy region[10]. Instability is evident for the case when $\theta_y$ is much greater than 1, indicating the *worst case* Taylor rule is one in which $\theta_y$ significantly exceeds $\theta_\pi$.

Figure 1: Left Determinacy region for parameters $\alpha = 1, \phi = 0.4$ , Right: Three dimensional plot of the determinacy region in $[\theta_\pi, \theta_y,\ loss]$ space



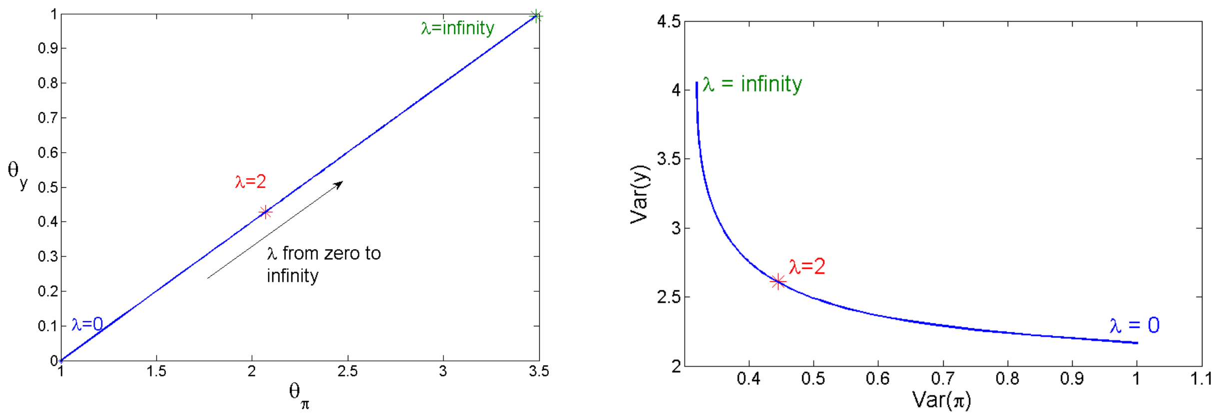

[9]Determinacy requires the eigenvalues of the system to be contained within the unit circle for stability. More generally, the number of eigenvalues within the unit circle must be equal to the number of free state variables. In this system, there are two state variables, $\pi_t$ and $\widetilde{y}_t$.

[10]To compute the loss, an IS curve shock was implemented in period 2, and the loss is computed for a total of 10 periods.

### 3.3.3 Optimal solution

The relative importance of inflation in the loss function, $\lambda$, can be varied to determine the optimal interest rate parameters $[\theta_\pi, \theta_y]$, which can lie anywhere on the line[11] (Figure 2, left) with $\lambda \to 0$ corresponding to strict output targeting, and $\lambda \to \infty$ to strict inflation targeting.

Figure 2: Left: Effect of increasing $\lambda$ from 0 to infinity on the optimal solution in $[\theta_\pi, \theta_y]$ space, Right: Efficiency frontier for the standard Taylor rule




Similarly, $\lambda$ can be varied to determine the output-inflation variance frontier, which is a map of points in the region $[Var(\pi_t), Var(\widetilde{y_t})]$(Figure 2, right). Monetary policy is defined as more efficient the closer the frontier is to the origin[12]. Intuitively, the relative variance of inflation is low when the central bank is an inflation targeter, and the variance of output is low when the central bank practices output targeting. The frontier can be used to compare alternative Taylor rules (see Ball, 1999b, Dennis, 2000 and De Brouwer and Regan, 1997).

# 4 Optimal Policy with Interest Rate Smoothing

## 4.1 State Space Representation

In this paper, control theory is applied to monetary policy, where the control variable is the interest rate set by the central bank. The optimal path of interest rates is set to minimise a welfare loss function subject to macroeconomic constraints. The state variables are governed by the economic constraints, the Phillips and IS curves, which trace the path of inflation and the output gap respectively.

[11]The optimal interest rate rule for $\lambda = 2$ are $\theta_\pi^* = 2.07$ and $\theta_y^* = 0.43$.

[12]To estimate the frontier, the variance of inflation and output is computed in response to a temporary IS curve shock in one period.

The macroeconomic constraints can be generalised in state space form, where $X_t = \begin{bmatrix} \pi_t & \widetilde{y}_t & i_{t-1} \end{bmatrix}'$. The state equation determines the path of all state variables contained in $X_t$.

$$X_{t+1} = AX_t + Bi_t + u_{t+1} \tag{4.1}$$

To determine the loss function, the state variable $X_t$ needs to be transformed into the output variable, $Y_t = \begin{bmatrix} \pi_t & \widetilde{y}_t & i_t - i_{t-1} \end{bmatrix}'$, which contains all variables included in the loss function in equation (3.3).

$$Y_t = C_x X_t + C_i i_t \tag{4.2}$$

The quadratic loss function takes the following form, with $K$ a $3 \times 3$ matrix with diagonal elements 1, $\lambda$ and $v$.

$$L_t = Y_t' K Y_t \tag{4.3}$$

Linear feedback instrument rules suggest that interest rates are linearly dependent on the state variable. The feedback control parameter $f$ must be chosen to minimise the loss function, yielding an optimal interest rate rule.

$$i_t = fX_t \tag{4.4}$$

The control problem is set as follows, by minimising a loss function with respect to the constraints of the state and control variables.

$$\begin{aligned} \underset{min}{L} &= \int_0^{\infty} Y_t' K Y_t dt \\ &subject\ to\ constraints \\ X_{t+1} &= AX_t + Bi_t + u_{t+1} \\ Y_t &= C_x X_t + C_i i_t \\ i_t &= fX_t \end{aligned} \tag{4.5}$$

## 4.2 Control solution

The aim of monetary policy is to minimise the losses of deviations of inflation, output and interest rate changes, $i_t - i_{t-1}$, controlling for interest rate smoothing. The relative preference for smoothing is governed by the parameter $v$.

$$L_t = \lambda \pi_t^2 + \widetilde{y}_t^{\,2} + v(i_t - i_{t-1})^2 \tag{4.6}$$

The optimal rule is a function of inflation, the output gap and the lagged interest rate, where $\rho$ represents the degree of interest rate smoothing.

$$i_t = \rho i_{t-1} + (1-\rho)\left[\theta_\pi \pi_t + \theta_y \widetilde{y}_t\right], \quad 0 < \rho < 1 \tag{4.7}$$

To solve for this model, control methods in Rudebusch and Svensson (1999) will be implemented[13].

[13] Formulation of solution is provided in the appendix.

The state equation is determined below, where the state variable is $X_t = \begin{bmatrix} \pi_t & \tilde{y}_t & i_{t-1} \end{bmatrix}'$.

$$\begin{bmatrix} \pi_{t+1} \\ \tilde{y}_{t+1} \\ i_t \end{bmatrix} = \begin{bmatrix} 1 & \phi & 0 \\ \alpha & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_t \\ \tilde{y}_t \\ i_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ -\alpha \\ 1 \end{bmatrix} i_t + \begin{bmatrix} 0 \\ u_{t+1} \\ 0 \end{bmatrix} \tag{4.8}$$

The output variable is $Y_t = \begin{bmatrix} \pi_t & \tilde{y}_t & i_t - i_{t-1} \end{bmatrix}'$, in equation (4.9).

$$\begin{bmatrix} \pi_t \\ \tilde{y}_t \\ i_t - i_{t-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} \pi_t \\ \tilde{y}_t \\ i_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} i_t \tag{4.9}$$

The loss function in matrix form is equal to $Y_t'KY_t$, where $K = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & v \end{bmatrix}$.

$$L_t = \begin{bmatrix} \pi_t & \tilde{y}_t & i_t - i_{t-1} \end{bmatrix} \begin{bmatrix} \lambda & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & v \end{bmatrix} \begin{bmatrix} \pi_t \\ \tilde{y}_t \\ i_t - i_{t-1} \end{bmatrix} = \lambda \pi_t^2 + \tilde{y}_t^2 + v(i_t - i_{t-1})^2 \tag{4.10}$$

The interest rate rule is given by $i_t = fX_t$, where $f$ is the matrix of control parameters.

$$i_t = \begin{bmatrix} f_1 & f_2 & f_3 \end{bmatrix} \begin{bmatrix} \pi_t \\ \tilde{y}_t \\ i_{t-1} \end{bmatrix} = f_1\pi_t + f_2\tilde{y}_t + f_3 i_{t-1} \tag{4.11}$$

Matrices $f$ and $V$ are determined by solving equations (7.11) and (7.12) simultaneously, where $Q$, $U$ and $R$ are as shown.

$$Q = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & v \end{bmatrix}, \quad U = \begin{bmatrix} 0 \\ 0 \\ -v \end{bmatrix}, \quad R = v \tag{4.12}$$

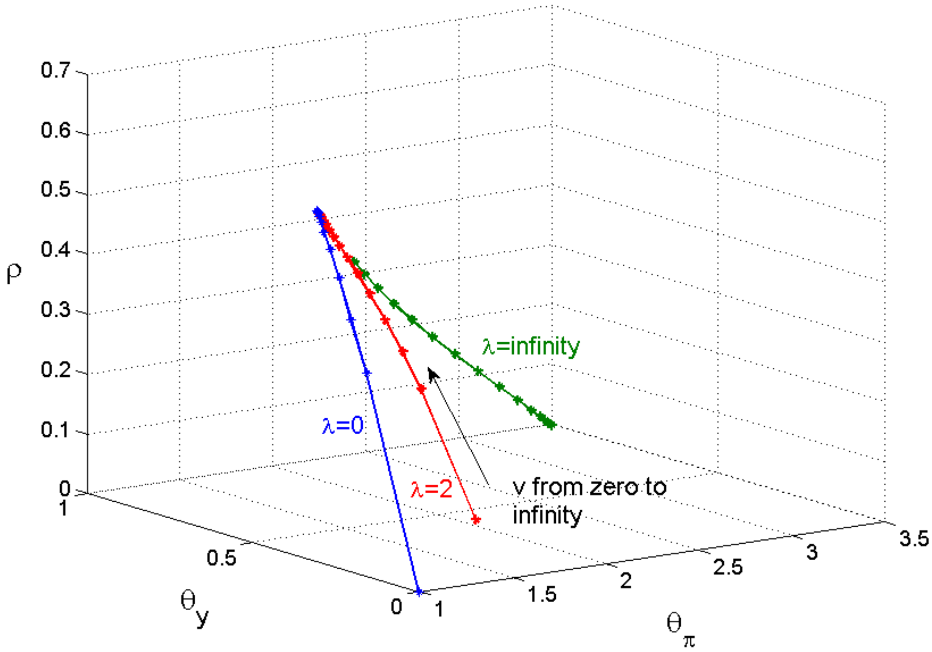
The control parameters $f_1$, $f_2$ and $f_3$ are a function of the parameters in the optimal interest rate rule in equation (4.7).

$$f_1 = (1-\rho)\theta_\pi \quad f_2 = (1-\rho)\theta_y \quad f_3 = \rho \tag{4.13}$$

## 4.3 Optimal solution

The effect of $v$ on the optimal solution in $[\theta_\pi, \theta_y, \rho]$ space is examined in Figure 3. As $v$ goes from zero to infinity, the central bank is more concerned with smoothing interest rates, and the lagged interest rate is more significant in the Taylor rule. For example, $\rho$ is zero when $v$ is zero, and rises to 0.586 as $v$ approaches infinity. The inflation and output gap coefficients, $\theta_\pi$ and $\theta_y$ , fall as $v$ rises, which means the central bank is less aggressive in stabilising inflation and output.

Figure 3: Optimal solution as $v$ increases from zero to infinity




The interest rate rules for combinations of $[\lambda, v]$ are summarised in Table 2.

Table 2: Interest rate rules for combinations of $[\lambda, v]$ in the central bank's loss function

| $[\lambda, v]$ | Interest rate rule | $\rho$ | $\theta_\pi$ | $\theta_y$ |
|---|---|---|---|---|
| $[2, 0]$ | $i_t = 2.07\pi_t + 0.43\widetilde{y_t}$ | 0.0 | 2.07 | 0.43 |
| $[2, 8]$ | $i_t = 0.4i_{t-1} + 0.6(1.53\pi_t + 0.44\widetilde{y_t})$ | 0.40 | 1.53 | 0.44 |
| $[2, \infty]$ | $i_t = 0.586i_{t-1} + 0.414(\pi_t + 0.306\widetilde{y_t})$ | 0.586 | 1.0 | 0.306 |

## 4.4 Effects of interest rate smoothing on optimal policy

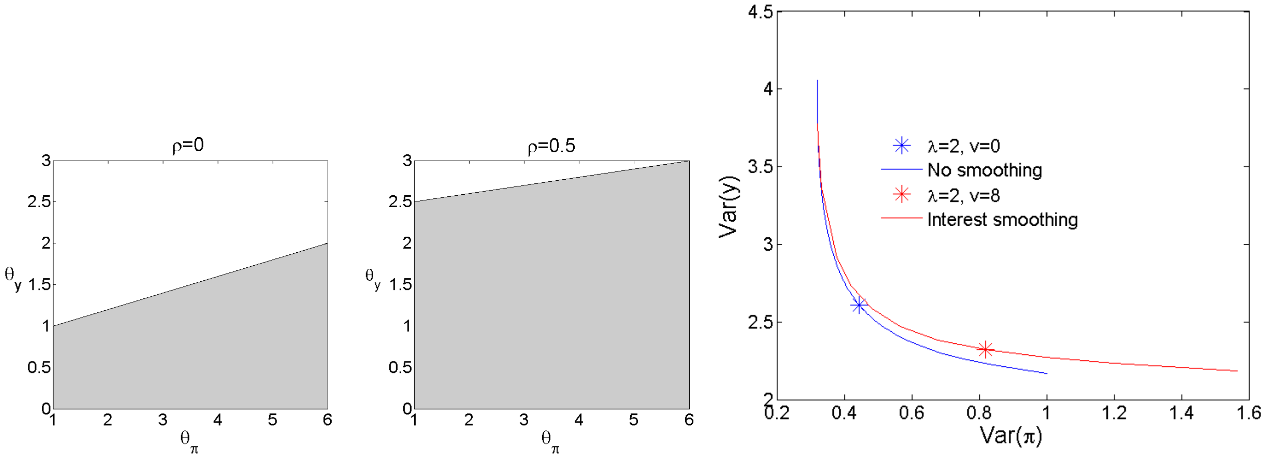
### 4.4.1 Determinacy region

The determinacy region can be computed using the eigenvalue method, where the new system is as follows,

$$\begin{bmatrix} \pi_{t+1} \\ \widetilde{y}_{t+1} \\ i_t \end{bmatrix} = \begin{bmatrix} 1 & \phi & 0 \\ \alpha(1-(1-\rho)\theta_\pi) & -\alpha(1-\rho)\theta_y & -\alpha\rho \\ (1-\rho)\theta_\pi & (1-\rho)\theta_y & \rho \end{bmatrix} \begin{bmatrix} \pi_t \\ \widetilde{y}_t \\ i_{t-1} \end{bmatrix} \tag{4.14}$$

The region in $[\theta_\pi, \theta_y]$ space is computed for a range of smoothing parameters, $\rho = \{0, 0.5\}$ (Figure 4, left). As the smoothing parameter increases, the determinacy region becomes larger, implying an increase in system stability.

Figure 4: Left: Determinacy region for the case of interest rate smoothing, Right: Comparison of efficiency frontiers with and without interest rate smoothing



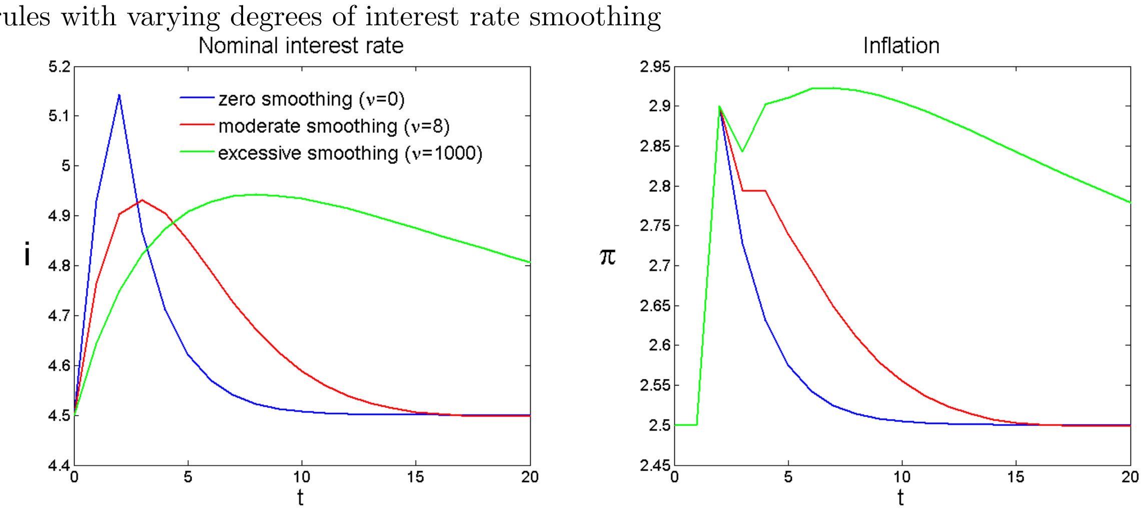

### 4.4.2 Output-inflation frontier

Comparing the frontiers for rules with and without interest rate smoothing (Figure 4, right), smoothing consists of higher combinations of output and inflation variance, and is less efficient. This is expected given interest rate smoothing represents a restriction on the use of the interest rate, which is the control variable.

### 4.4.3 Impulse response

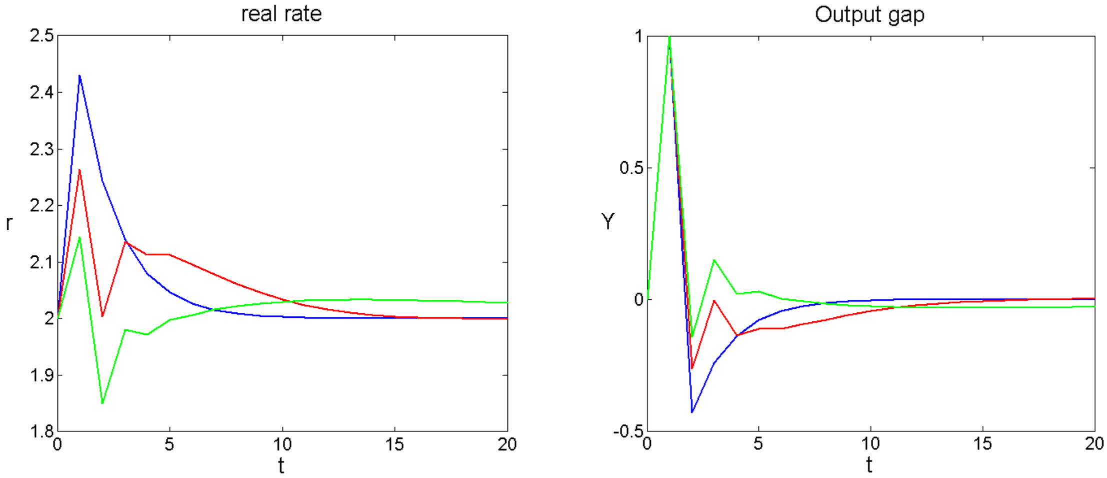
The time path of the interest rate and inflation in response to a one period output shock is shown in Figure 5. The trajectory of interest rates is less steep under interest rate smoothing, but in doing so causes output and inflation to stabilise over a longer time horizon. Placing restrictions on the control variable will make a system take longer to converge to equilibrium, and is a practical tradeoff encountered in policy when considering the extent of interest rate smoothing.

Figure 5: Impulse responses of the nominal interest rate and inflation with respect to an output shock, for rules with varying degrees of interest rate smoothing



The time path of the real rate and the output gap reveal some useful properties of interest rate smoothing (Figure 6). The real rate[14] under smoothing peaks less, and the under-shooting of output below potential is dampened. Reducing the peak and volatility of the real interest rate is a key advantage of smoothing, and supports the argument that smoothing can reduce disruption in financial markets[15] (Goodfriend, 1991).

Figure 6: Impulse responses of an output shock for the real interest rate and output gap




## 5 Characterising RBA interest rate settings

### 5.1 Methodology

Empirical regression methods (Clarida et al, 1998 and De Brouwer and Gilbert, 2005) are applied to characterise monetary policy in Australia as an interest rate rule. The equilibrium interest rate $i_t^*$ is set as follows, where $(E_t[\pi_{t+k}] - \pi^T)$ and $(E_t[y_{t+k} - y_{t+k}^n])$ measure the expectation of inflation and output deviations respectively.

$$i_t^* = r_n + \theta_\pi(E_t[\pi_{t+k}] - \pi^T) + \theta_y(E_t[y_{t+k} - y_{t+k}^n]) \tag{5.1}$$

The value of $k$ can range from 1 quarter to 4 quarters. The adjustment of the actual interest rate $i_t$ towards $i_t^*$ is governed by the degree of smoothing, $\rho$.

$$i_t = \rho i_{t-1} + (1-\rho) i_t^* \tag{5.2}$$

Substituting the equilibrium interest rate in equation (5.1) into equation (5.2) yields,

$$i_t = a + \rho i_{t-1} + b_\pi E_t[\pi_{t+k}] + b_y E_t[y_{t+k} - y_{t+k}^n] \tag{5.3}$$

[14] For the impulse responses, it is assumed that the equilibrium rate of inflation is 2.5 per cent, and the equilibrium real interest rate is 2 per cent.

[15] Goodfriend, 1991 suggests that interest rate smoothing cushions the banking system against interest rate shocks. For example, as banks make loan contracts prior to an interest rate change, there exists negative correlation between competitive deposit rates and bank profits. Unanticipated rate shocks can create negative cash flows and bank insolvencies.

This is a forward looking rule, in that policy responds to information about future values of inflation and the output gap. Alan Greenspan, the former Fed Chairman, acknowledges that since monetary policy works with a lag, it is based on conditions likely to prevail six to twelve months ahead (Orphanides, 2001). In order to obtain realised values of inflation and the output gap, expectational errors are added to equation (5.3).

$$i_t = a+\rho i_{t-1}+b_\pi \pi_{t+k}+b_y(y_{t+k}-y^n_{t+k})-b_\pi(\pi_{t+k}-E_t[\pi_{t+k}])-b_y\left\{y_{t+k}-y^n_{t+k}-E_t[y_{t+k}-y^n_{t+k}]\right\}) \quad (5.4)$$

The expectational errors are equivalent to a residual $u_t$, which is assumed to have a distribution, $u_t \sim N(0,\sigma^2)$.

$$u_t = -b_\pi(\pi_{t+k}-E_t[\pi_{t+k}])-b_y\left\{y_{t+k}-y^n_{t+k}-E_t[y_{t+k}-y^n_{t+k}]\right\} \quad (5.5)$$

The residual $u_t$ is correlated with the explanatory variables $\pi_{t+k}$ and $y_{t+k}$, which requires equation (5.6) to be estimated via method of moments with instrumental variables.

$$i_t = a+\rho i_{t-1}+b_\pi \pi_{t+k}+b_y(y_{t+k}-y^n_{t+k})+u_t \quad (5.6)$$

The instrumental variables are, in accordance with De Brouwer and Gilbert (2005), lags of the interest rate, inflation and the output gap.

## 5.2 Data sources

### 5.2.1 Headline and underlying inflation

Dhawan and Jeske (2007) suggest that underlying or core measures of inflation are more appropriate to use in estimating a Taylor rule, as it captures the general trend in inflation over a given period (Figure 7, left). The RBA implicitly targets underlying measures of inflation as it trims volatile or temporary price shocks[16] (Stevens, 2008).

### 5.2.2 Output gap

The output gap is defined as the percentage deviation of actual output from potential output. The output gap is estimated via fitting a linear trend to the log of real GDP, $Y_t$, where $t$ is a time trend, and $\beta$ is an approximate annual growth rate, and $v_t$ is an error term. The output gap is the difference between $logY_t$ and trend output estimated in equation (5.7).

$$logY_t = \alpha+\beta t+v_t \quad (5.7)$$

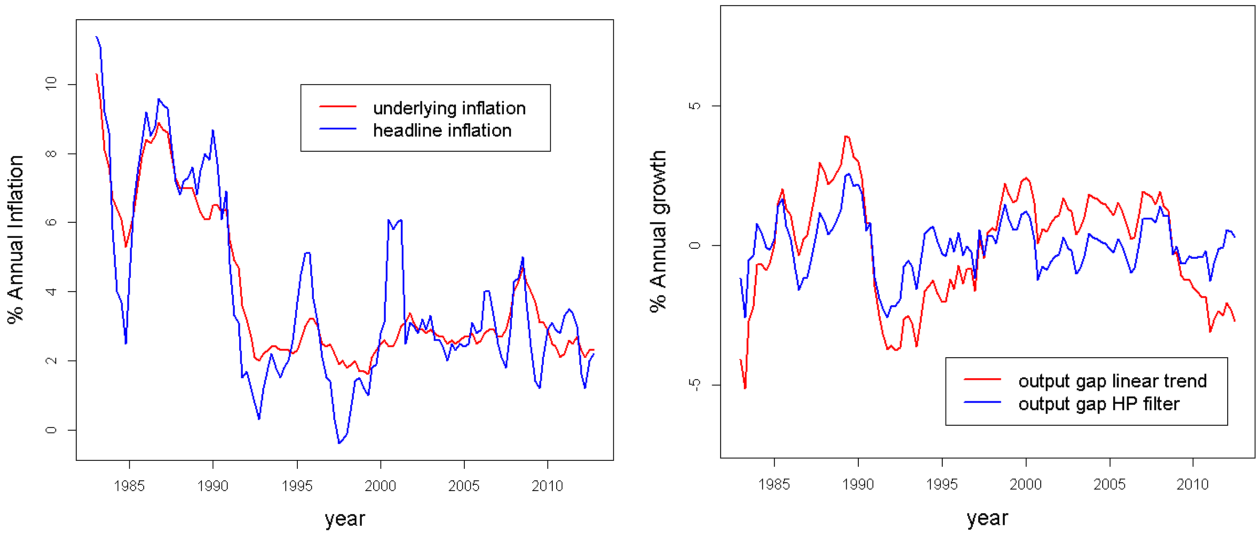
[16]Underlying inflation is calculated as a trimmed mean, which disregards the bottom and top 15 per cent of price changes in a CPI basket. For example, a temporary shock to inflation in 2000 due to the onset of GST is not recorded in underlying measures.

An alternative method to determine the output gap is the Hydrick Prescott (HP) Filter, which determines a trend component of GDP, $g_t$, by minimising the following function, where $y_t$ is the log of GDP.

$$L = \sum_{t=1}^{T}(y_t - g_t)^2 + \lambda \sum_{t=2}^{T-1}[(g_{t+1} - g_t) - (g_t - g_{t-1})]^2 \qquad (5.8)$$

The HP filter attempts to smooth deviations in trend, where $\lambda$ is the degree of smoothing[17]. The output gap method estimates (Figure 7, right) are of different sign in the period 2000 to 2005.

Figure 7: Left: Headline and underlying inflation for Australia, 1983 - 2012, Right: Output gap for Australia, 1983-2012




Source: RBA statistics (2012)

### 5.3 Estimation Results

The estimated regression equation (5.6) takes the following form as a Taylor rule, where $b_\pi = (1-\rho)\theta_\pi$ and $b_y = (1-\rho)\theta_y$. Estimates of $b_\pi$, $b_y$ and $\rho$ yield estimates of the inflation and output gap coefficients, $\theta_\pi$ and $\theta_y$. This allows us to empirically evaluate to what extent the RBA has operated on the basis of a Taylor rule[18] (Table 3).

$$i_t = \rho i_{t-1} + (1-\rho)[r_n + \theta_\pi(\pi_{t+k} - \pi^T) + \theta_y(y_{t+k} - y^n_{t+k})] \qquad (5.9)$$

The interest rate smoothing parameter $\rho$ ranges from 0.7 to 0.8 and is significant for all forward looking rules. This indicates a high degree of inertial behaviour in setting interest rates. The inflation coefficient is one when using present values of inflation, and rises to 1.87 when based on inflation three

[17] A $\lambda$ of 1600 is recommended for quarterly data.
[18] Underlying inflation and linear trend output gap estimates are used int he analysis.

quarters ahead. On the contrary, the output gap parameter falls as monetary policy becomes more forward looking. Forward looking rules ($k = 1, 2, 3$) satisfy the Taylor principle, $\theta_\pi > 1$, which is required for stability of inflation. Empirically, rules follow the Taylor principle only when dependent on forward looking inflation and output gap data (Clarida et al, 1998 and De Brouwer and Gilbert, 2005).

Table 3: Estimation results for interest rate rules with different time horizons

| Time horizon | $a$ | $\rho$ | $b_\pi$ | $b_y$ | $\theta_\pi$ | $\theta_y$ |
|---|---|---|---|---|---|---|
| $k = 0$ | 0.52 | 0.85** | 0.16 | 0.21** | 1.00 | 1.35* |
| | (0.27) | (0.06) | (0.13) | (0.07) | (0.51) | (0.64) |
| $k = 1$ | 0.38 | 0.77** | 0.35* | 0.17* | 1.52** | 0.73* |
| | (0.27) | (0.06) | (0.14) | (0.07) | (0.29) | (0.36) |
| $k = 2$ | 0.27 | 0.73** | 0.46** | 0.11 | 1.73** | 0.41 |
| | (0.27) | (0.06) | (0.14) | (0.07) | (0.25) | (0.30) |
| $k = 3$ | 0.16 | 0.72** | 0.53** | 0.05 | 1.87** | 0.15 |
| | (0.27) | (0.06) | (0.14) | (0.07) | (0.23) | (0.27) |

Note: *,** indicate statistical significance at the 5 and 1 per cent level respectively
Standard errors of $\theta_\pi$ and $\theta_y$ are calculated using the delta method

## 5.4 Comparison of optimal rules with RBA policy

The historical analysis of monetary policy yields estimates of inflation and output gap coefficients that are consistent with optimal rule parameters (Table 4).

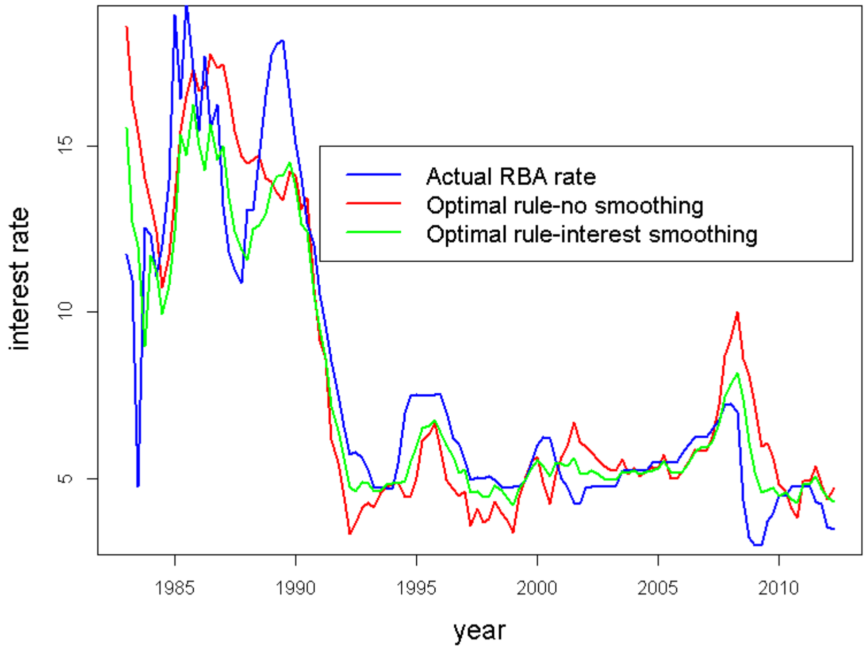
Table 4: Summary of optimal rules for setting RBA interest rates, $r_n = 2.5$ and $\pi^T = 2.5$

| $[\lambda, v]$ | Interest rate rule | $\rho$ | $\theta_\pi$ | $\theta_y$ |
|---|---|---|---|---|
| $[2, 0]$ | $i_t = 2.5 + \pi_t + 1.07(\pi_t - 2.5) + 0.43\widetilde{y}_t$ | 0 | 2.07 | 0.43 |
| $[2, 8]$ | $i_t = 0.4i_{t-1} + 0.6(2.5 + \pi_t + 0.53(\pi_t - 2.5) + 0.44\widetilde{y}_t)$ | 0.4 | 1.53 | 0.44 |

The critical difference is in the smoothing parameter $\rho$, which is 0.77 for the forward looking rule, and is larger than the optimal value of $\rho$, which can range from 0 to 0.6. This suggests that RBA interest rates are too inertial in practice, to the extent that it is sub-optimal[19]. The RBA is averse to interest rate volatility in order to prevent disruption of financial markets (Goodfriend, 1991), where fluctuation in interest rates may trigger uncertainty in the investment decisions of households, businesses and the financial sector. Large interest rate changes are also politically hard to achieve, and may signal incompetence to the public.

[19] A critical assumption is that optimal rates are set in each quarter, in order to synchronise with quarterly data. If the optimal rule is in fact based on annual rate settings, then the quarterly rate of smoothing is equal to $\rho = \sqrt[4]{0.40} = 0.80$, which is consistent with the excessive rate of smoothing practiced by the RBA.

Figure 8: Interest rates set by RBA relative to a Taylor rule




In Figure 8, the RBA cash rate is compared to optimal rules. The interest rate smoothing optimal rule more closely follows the actual path of rate settings. For example, in 2008, the cash rate reached a peak of 7.25 per cent in response to high inflation[20]. The optimal rule incorporating smoothing suggests the interest rate should peak at 8 per cent, as opposed to 10 per cent for a rule with zero smoothing.

An important question that arises is to what extent optimal rules can be used in formulating and forecasting monetary policy decisions. Comparisons of optimal rules with RBA policy are subject to the *Lucas critique* (Lucas, 1976), in which policy evaluations based on macroeconomic models are subject to uncertainty. Using an optimal rule to formulate policy is constrained by technical limitations of the Taylor rule. For example, systematic differences between the optimal rule and the actual RBA rate may reflect shocks to the natural rate of interest, assumed at 2.5 per cent in the analysis[21].

# 6 Conclusion

This paper examined an analytical and numerical solution for optimal monetary policy in a Keynesian model based on Ball (1999a). The optimal rule adheres to the Taylor principle, in which the interest

[20]Underlying inflation peaked at 4.7 per cent in 2008

[21]The natural rate of interest, $r_n$, can be subject to shocks such as a change in total factor productivity. A limitation of standard Taylor rules is the assumption of a constant rate of interest. $r_n = 2.5$ is based on the observed average real rate of 2.5 per cent in the period 1996 to 2012.

rate responds to inflation by more than a one-for-one basis. A central bank that practices strict inflation targeting require more aggressive inflation and output gap coefficients in the rule.

In practice, monetary policy exhibits a degree of interest rate smoothing, and this has profound implications for optimal policy. For example, as the lagged interest rate becomes more significant, the determinacy region expands, implying an increase in system stability. When a central bank is more concerned with smoothing interest rates, the lagged interest rate becomes more significant in the optimal rule, with lower inflation and output gap parameters. Impulse responses to an output shock show lower peaks and variance in the real rate and output gap when there is smoothing. Stability in the real rate is useful, as it can reduce uncertainty in financial markets, and in business and consumption decisions. However, the tradeoff of smoothing is a loss of efficiency and a longer time horizon to stabilise inflation and output.

Characterising RBA interest rate settings as a Taylor rule suggest estimates of inflation and output gap coefficients are consistent with optimal rules. The major difference is in the smoothing parameter, which is equal to 0.77, and is sub-optimal. Despite the technical issues that arise in using optimal rules, they can still be used to guide and evaluate policy decisions. The analysis provides a compelling case for the RBA to practice lower levels of interest rate smoothing, and accordingly move rates more aggressively in response to inflation and output deviations from their equilibrium values.

# 7 Appendix

## 7.1 Analytical solution

In controlling the interest rate at time $t$, the central bank takes one period to affect the output gap, $\widetilde{y_{t+1}}$, and takes two periods to affect inflation, $\pi_{t+2}$. Policy aimed at stabilising inflation is in effect targeting the expectation of inflation two periods hence. Therefore, optimal policy suggests a linear relationship between $E_t(\widetilde{y_{t+1}})$ and $\pi_{t+1}$, with the parameter $q$ to be determined.

$$E_t(\widetilde{y_{t+1}}) = -q\pi_{t+1} \tag{7.1}$$

Using this relationship, equation (3.7) can be rewritten to state the output gap is linearly related to inflation.

$$\widetilde{y_t} = E_{t-1}(y_t) + u_t = -q\pi_t + u_t \tag{7.2}$$

The period loss function is a linear weighting of the variances of inflation, $E[\pi_t^2]$ and the variance of the output gap, $E[\widetilde{y_t}^2]$. The relative importance of inflation is given by the weight $\lambda$. In this loss function, it is assumed that the equilibrium inflation rate is zero.

$$L_t = \lambda E[\pi_t^2] + E[\widetilde{y_t}^2] \tag{7.3}$$

To derive the inflation variance, let us substitute $\widetilde{y_t}$ in equation (7.2) into the Phillips curve in equation (3.4). Inflation in period $t+1$ is now a function of inflation in the previous period.

$$\pi_{t+1} = \pi_t(1 - \phi q) + u_t \tag{7.4}$$

For determinacy and a stable equilibrium, let us assume the variance of inflation is equal in each period, so $E[\pi_{t+1}^2] = E[\pi_t^2]$. Given the variance of the exogenous IS shock is $\sigma_{u_t}^2$, the variance of inflation can be determined as follows.

$$E[\pi_t^2] = \frac{\sigma_{u_t}^2}{(1 - (1 - \phi q)^2)} \tag{7.5}$$

The variance of the output gap, using equation (7.2), can be determined to be a function of inflation variance.

$$E[\widetilde{y_t}^2] = q^2 E[\pi_t^2] + \sigma_{u_t}^2 \tag{7.6}$$

Substituting inflation and output variance into equation (7.3) yields the loss function in terms of $\lambda$, $q$, $\phi$ and the shock variance $\sigma_{e_t}^2$.

$$L_t = \frac{(\lambda + q^2)\sigma_{u_t}^2}{(1 - (1 - \phi q)^2)} + \sigma_{u_t}^2 \tag{7.7}$$

Minimising the loss function with respect to $q$ will yield the optimal $q^*$, which is a function of the Phillips curve parameter $\phi$ and the inflation sensitivity $\lambda$.

$$\frac{\partial L_t}{\partial q} = 0 \Rightarrow q^* = \frac{-\phi\lambda + \sqrt{\phi^2\lambda^2 + 4\lambda}}{2} \tag{7.8}$$

The optimal $q$ can be used to derive the optimal interest rate rule. The linearity between $E_t(y_{t+1})$ and $\pi_{t+1}$ established in equation (7.1) enables the interest rate $i_t$ to be a function of the state variables $\pi_t$ and $\widetilde{y_t}$.

$$E_t(\widetilde{y_{t+1}}) = -\alpha(i_t - \pi_t) = -q\pi_{t+1} = -q(\pi_t + \phi\widetilde{y_t}) \tag{7.9}$$

Rearranging equation (7.9) and substituting in $q^*$ yields the optimal Taylor rule.

## 7.2 Control solution

To solve for the control parameter $f$, the Hamiltonian needs to be constructed, and differentiated with respect to $i_t$, $X_t$ and $\lambda$ to derive the control, costate and state equations [22].

[22] The control equation is $\frac{\partial H}{\partial i_t} = 0$, the costate equation is $\frac{\partial H}{\partial X_t} = -\dot{\lambda}_t$ and the state equation is $\frac{\partial H}{\partial \lambda_t} = X_{t+1}$.

$$H = Y_t' K Y_t + \lambda_t (AX_t + Bi_t + u_{t+1}) \tag{7.10}$$

The optimal control parameter $f$ is a function of the matrices $R$, $B$, $V$, $A$ and $U'$ (Rudebusch and Svensson, 1999a).

$$f = -(R + B'VB)^{-1}(U' + B'VA) \tag{7.11}$$

The matrix $V$ is determined by the algebraic Ricatti equation and is used as an input in order to determine matrix $f$.

$$V = Q + Uf + f'U' + f'Rf + M'VM \tag{7.12}$$

$$Q = C_x' K C_x, \;\; U = C_x' K C_i, \;\; R = C_i' K C_i \tag{7.13}$$

Solving equations (7.11) and (7.12) simultaneously will determine $f$.

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