ACTUARIAL SCIENCE HONOURS RESEARCH REPORT

Curtin University

Arlene Janse van Rensburg

Supervised by Mr Mark Hayes and Dr Aloke Phatak

COMPARING RISK BASED REGULATORY CAPITAL REQUIREMENTS FOR MARKET RISK USING COPULAS

arlene.jansevan@student.curtin.edu.au

20774518

Period of submission: Semester 2, 2025

Table of Contents

Acknowledgements

I would firstly like to thank my supervisors, Mr Mark Hayes and Dr Aloke Phatak, for their invaluable expertise, patience and encouragement over the past year. I appreciate the time they have generously dedicated to providing advice, reviewing my work, and helping me to explore the application of copulas in an actuarial and regulatory context. This thesis would not have been possible without their guidance.

I am also deeply grateful to my parents, Carla and Antoin, for their unwavering support and love during this year and the rest of my studies. Thank you for everything.

Finally, I would like to thank the Australian Prudential Regulation Authority (APRA) and the Reserve Bank of Australia (RBA) for generously awarding me the Brian Gray Scholarship. The financial support and access to additional expertise has greatly assisted my research efforts.

List of figures

List of tables

List of equations

Abstract

This report investigates how alternative dependence modelling approaches influence the assessment of market risk capital requirements for life insurers. Current regulatory frameworks, including the Australian Prudential Regulation Authority (APRA) Standard Method and comparable international regimes, determine market risk capital using stress-based charges aggregated through fixed correlation matrices calibrated to achieve a 99.5% probability of sufficiency over one year. While this approach is widely applied, it relies on simplifying assumptions regarding dependence between asset classes.

Using a notional Australian life insurer balance sheet, this research report applies copula-based joint modelling to equity, property, and fixed income asset returns to examine the sensitivity of 1-in-200 year adverse joint surplus outcomes under different dependence structures. Gaussian, Student's t, and Clayton copulas are fitted to historical return data and used to simulate joint surplus distributions, enabling the explicit consideration of tail dependence. To address the limitation of a short annual return history, a reshuffling procedure is introduced in which historical monthly returns are rearranged into alternative but equally plausible annual sequences, with copula parameters re-estimated for each sequence.

The results indicate that copula-based modelling can lead to materially different estimates of market risk capital compared to the Australian Standard Method, particularly when asymmetric dependence is assumed. The reshuffling analysis further demonstrates that the estimated 1-in-200 year adverse outcomes are sensitive to historical sequencing and dependence parameter uncertainty. Overall, the findings highlight the role of dependence assumptions in the determination of market risk capital requirements and illustrate how copula-based methods can be used as an addition to or exploratory tool to complement existing regulatory approaches.

1. Introduction

Life insurance plays a vital role in Australia, particularly as the aging population places increasing strain on government budgets. In the year ending December 2024, the Australian life insurance industry paid out AUD13.468 billion in claims. Statutory funds, which are dedicated pools of assets designed to protect policyholders, held AUD127.8 billion in total assets (APRA, 2025). Maintaining public trust in the financial resilience of life insurers is essential, especially as Australians aged over 65 are projected to comprise over 21% of the population by 2066 (AIHW, 2024). A larger retiree population will rely more heavily on life insurers for income, annuities, and protection products, making confidence in insurers' long-term solvency critical to financial security and social stability.

To ensure financial resilience, regulators impose capital requirements that insurers must meet to guard against financial stress. In Australia, the Australian Prudential Regulation Authority (APRA) defines capital adequacy standards for life insurers, representing the minimum capital required to withstand a 1-in-200-year adverse event over a one-year period. This can also be interpreted as a 99.5% probability of remaining solvent over the next financial year.

APRA sets out a "standard method" to calculate a life insurer's required capital. While this framework ensures consistency and comparability, it may oversimplify the complex dependencies between risk components, especially the relationship between different asset classes in an insurer's balance sheet. Alternative approaches, such as copula-based models, may capture tail dependencies and asymmetric relationships between asset risk components more accurately. These methods are increasingly recognised in academic literature as more risk-sensitive and potentially more efficient in determining capital requirements. This thesis will investigate whether the current regulatory approach to aggregating the Asset Risk charge components sufficiently captures the risks posed by market risk, and whether alternative statistical methods could lead to similar, more accurate or more economically efficient capital outcomes.

2. Research questions

The aim of this thesis is to examine and compare different approaches for determining regulatory capital requirements for market risk in the life insurance industry, with a focus on the Australian prudential framework and global best practises. I will investigate whether alternative statistical approaches can provide a more risk-sensitive assessment of capital requirements for market risk compared to the current regulatory framework.

The specific aims will be addressed through three research questions:

1. How are regulatory capital requirements for market risk in life insurance currently determined in Australia and globally?
2. Which alternative methods can be used to analyse the interaction between variables affecting market risk to enable us to explore tail behaviour?
3. Does using a copula-based joint distribution significantly alter the regulatory capital requirements for market risk compared to current regulatory approaches?

3. Significance

Life insurance plays an important role in providing financial protection to policyholders in need. Capital adequacy regulations support individual policyholders and broader financial stability by aiming to ensure that insurers remain solvent during periods of financial stress. This thesis will investigate whether alternative risk modelling techniques can enhance the risk sensitivity of capital requirements, which will contribute to the ongoing refinement of regulatory frameworks governing market risk in the life insurance sector. A more risk sensitive quantification of market risk exposure could improve capital allocation, reducing both undercapitalization and excessive capital buffers.

The prudential capital requirements for life insurers are of particular concern to the Australian Prudential Regulation Authority (APRA) and the actuarial profession in Australia. In February 2025, APRA confirmed its intention to engage in a public consultation on capital requirements for annuity products. The key proposed change aims to lower life insurance capital requirements for annuity products and support life insurers to provide more products for retirees (APRA, 2025). Through these consultations, APRA is recognising that the current calculation of capital requirements is potentially too onerous.

Therefore, this thesis' investigation focussing on the calculation of capital requirements for market risk supports APRA's wider steps towards improving the efficiency of capital allocation for Australian life insurers while maintaining financial security.

Sustained heightened volatility and uncertainty in financial markets also poses significant challenges to institutional investors such as life insurers. Effective management of market risk in Australia is critical for both the solvency of individual firms and the stability of the broader financial system. An international example of the perils of market risk for insurers was the near failure of insurance giant American International Group (AIG) in the 2008 global financial crisis, which required a significant government bailout to avoid collapse and a systemic crisis. The near failure was largely attributed to AIG's failure to manage the market risks associated with its massive credit default swap business (McDonald & Paulson, 2015). Domestically, HIH Insurance was Australia's second-largest insurance company before it was placed into provisional liquidation on 15 March 2001. HIH's failure had profound and far-reaching effects, impacting some two million policyholders and 1,000 employees (Campbell, 2018). Although this failure was caused by improper risk pricing and a failure to reserve properly for future claims, rather than inappropriate management of market risk (Campbell, 2018), the lasting and extensive aftermath in Australia demonstrates the importance of ensuring Australian regulations are appropriately geared towards avoiding other insurer collapses in the future.

Literature Review

The following sections will compare the current regulatory approaches to determining the capital requirements for market risk for life insurers in different jurisdictions. Since the modelling component of this thesis will focus specifically on the Australian prudential framework, the Australian approach to capital requirements is discussed first and the most comprehensively. This literature review does not cover a historical timeline of development within and between jurisdictions but rather compares the similarities and differences between the Australian and global framework's treatment and aggregation of market risk.

4. Australian approach to capital requirements

4.1 Background

The Life Insurance Actuarial Standards Board (LIASB) first introduced solvency and capital adequacy standards for life insurers in 1995, which were a requirement of the *Life Insurance Act 1995* (Life Act). The Life Act was amended in 2007 and transferred the responsibility for setting and administering prudential standards relating to solvency and capital adequacy to the Australian Prudential Regulation Authority (APRA) (APRA, 2010).

As the prudential regulator of the Australian financial services industry, APRA establishes and enforces standards and practices on institutions to protect the financial interests of the Australian community. APRA's role is to ensure the institutions they supervise are financially stable and can meet their obligations to depositors, policyholders, and members (APRA, 2010). APRA's principal objective in its regulation of life insurers is to protect policyholders. Capital is the foundation of financial strength and is a core element of APRA's regulatory framework (APRA, 2010).

In May 2010, APRA released a discussion paper outlining its proposals to update the capital standards for life insurers and general insurers. APRA's revised capital framework took effect from 1 January 2013.

APRA's intention in updating the standards was to make its capital requirements simpler, more risk sensitive, and to improve the alignment of its capital standards across the industries it regulates. APRA did not have the view that existing capital requirements for life insurers were too high or too low, and the aim was not for the new capital framework to result in higher capital requirements for life insurers. APRA intended to develop capital requirements that more appropriately reflect the risks undertaken both at an individual insurer level and by the life insurance industry as a whole (APRA, 2011).

APRA's new framework introduced the concept of a "capital base" for life insurers. The updated framework involves a single measure of required capital for life insurers which is compared with the life insurer's capital base.

4.2 Capital adequacy

A life insurer needs sufficient capital resources to be able to absorb unexpected losses and to manage other adverse shocks so that it can meet its commitment to policyholders. The Australian Prudential Regulation Authority (APRA) uses four primary concepts to assess capital adequacy (APRA, 2010):

- required capital is the minimum capital that APRA requires an insurer to hold,
- eligible capital is the capital held by an insurer which APRA recognises as eligible for capital adequacy purposes (the "capital base" for life insurers),
- surplus capital is the excess of eligible capital over required capital, net of any adjustments, and
- target surplus is the targeted amount of surplus capital as determined by the board of the regulated entity or group.

A life insurer is required to hold admissible assets that exceed the sum of its liabilities and its required capital. In Figure 1, this is shown as "surplus".

Target surplus aims to ensure that a life insurer's capital base does not fall below required capital. Insurers must set appropriate target surplus levels, and falling below these levels could trigger supervisory attention.

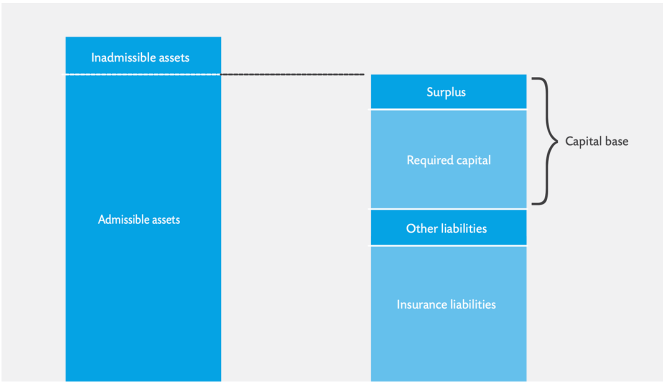

Figure 1: Structure of a life insurer's capital base (Source: Figure 3, APRA May 2010 discussion paper)

4.3 The PCR, PCA and asset risk charge

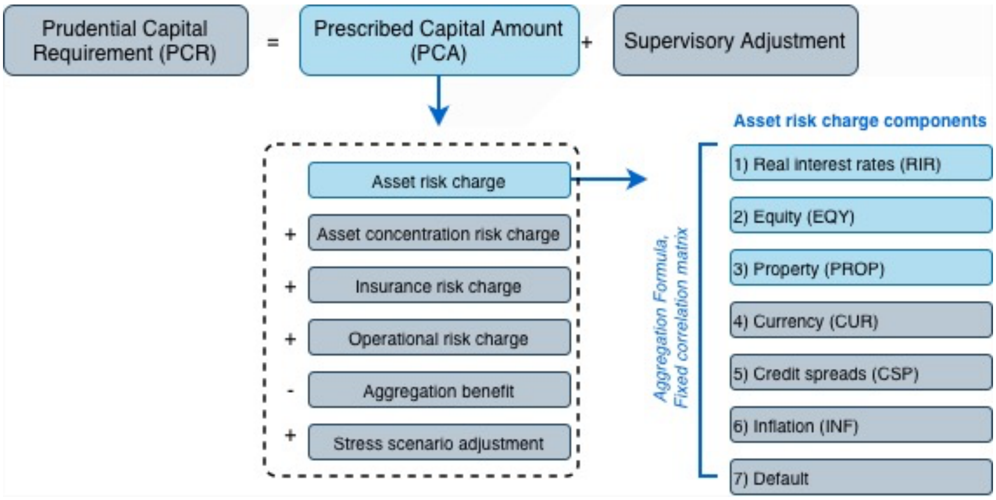

Figure 2: Overview of the Australian PCA and asset risk charge. (Created by author)

The prudential capital requirement (PCR) is the level of capital required by life insurers for regulatory purposes. It is intended to account for the full range of risks to which a fund or life company is exposed. The PCR is the sum of a prescribed capital amount (PCA) plus any supervisory adjustment determined by the Australian Prudential Regulation Authority (APRA). Prudential Standard LPS 110 (APRA, 2013) sets out APRA's "standard method" to calculate the PCA. As outlined on page 46 of APRA's May 2010 discussion paper about the asset risk charge, life insurers can also apply for approval to adopt an internal model-based method, provided they can demonstrate that it is embedded in their management, operations and decision-making processes. However, this thesis is specifically concerned with the standard method approach.

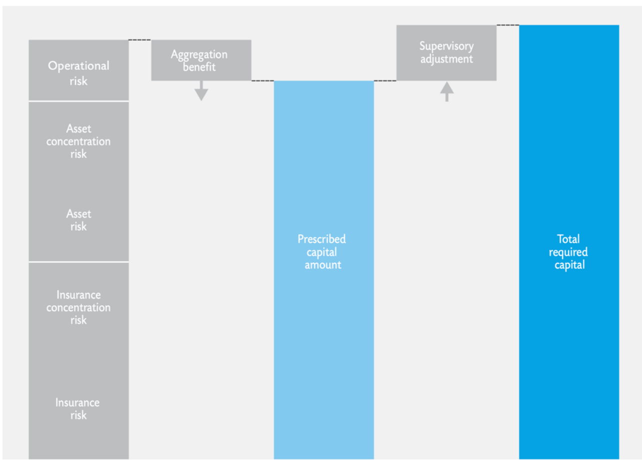

Figure 3: The structure of a life insurer's PCA and total required capital. (Source: APRA, 2010. Figure 4)

The PCA is intended to ensure a 99.5% probability that a life insurer will have sufficient capital to meet all obligations over a one-year reporting period (APRA, 2013). Insurers are required to hold enough capital to absorb nearly all unexpected adverse events, excluding only the most extreme 0.5% of scenarios, and still remain able to meet commitments to policyholders at the end of the reporting period. As discussed on page 21 of APRA's 2010 discussion paper, all risk assessments at this 99.5 per cent probability level are very approximate, and in some cases subjective, such that the ultimate details of the capital calculations are a matter of judgement for APRA.

The PCA for a fund is determined as the sum of individual risk charges, as illustrated in Figure 3. These are aggregated using an algorithm specified by APRA.

To isolate capital requirements for market risk, this thesis will investigate the asset risk charge, which captures the potential adverse impact on a fund's capital base arising from movements in the values of investments due to market or credit risk. The asset risk charge is concerned with a fund's entire capital base, meaning it depends on the total amount of admissible assets and how they are invested (APRA, 2013). A statutory fund that has its surplus capital invested in risky assets would have a higher asset risk capital requirement than an otherwise identical fund that invests its surplus capital more conservatively. This is because investment in risky assets leads to a mismatch of a fund's assets and liabilities and increases the volatility of assets supporting required capital and surplus.

As illustrated in Figure 2, the asset risk charge consists of seven risk charge components; Real Interest Rates (RIR), Expected Inflation (INF), Currency (CUR), Equity (EQY), Property (PROP), Credit Spreads (CSP), and Default. The risk charge components are not exhaustive, but APRA considers these to be the major risks likely to be encountered by insurers in the normal course of business (APRA, 2013). The required capital for each risk component is determined by applying stresses to the insurer's balance sheet, which are referred to as risk modules. Each component is assessed within its individual risk module, where the stresses are calibrated such that the resulting capital requirement for that module provides a 99.5 per cent probability of sufficiency over a one-year period (APRA, 2023). The capital charge for each risk module represents the reduction in the capital base relative to the unstressed position, with a minimum value of zero (APRA, 2023).

The individual capital charges are combined using an aggregation formula and a correlation matrix specified by APRA. The matrix is intended to account for the diversification effects when determining the total asset risk capital requirement (APRA, 2013). The result of applying the formula is defined as the aggregate risk charge component.

4.4 Stresses applied to the asset risk charge

The capital requirement for each asset risk component must be non-negative. If a stress does not reduce the life fund's capital base, the corresponding capital requirement for that risk module is set to zero. The stress scenarios for each risk module is set out in prudential standard LPS 114 and outlined below.

The stresses outlined in each of the risk charge components were determined by the Australian Prudential Regulation Authority (APRA) after considering relevant historic experience, both in Australia and internationally, and economic theory (APRA, 2011). As stated in APRA's 2010 Technical Paper (APRA, 2010), a "substantial degree of judgement was required".

Real interest rates stress

This module measures the impact on a fund's capital base from changes in real interest rates. Real interest rates are the portion of the nominal risk-free interest rates that remain after deducting expected CPI inflation. Assets and liabilities whose values are dependent on real or nominal interest rates, including fixed interest assets and insurance liabilities valued using a discount rate, must be revalued using the stressed real or nominal rates (APRA, 2023).

Table 1: Asset risk charge, real interest rate stress

Upward stress adjustment	$\text{upward stress} = \max(3\%\,; nominal\ risk\ free\ interest\ rate) \times\ 0.25$ The upward stress adjustment must be between 75 and 200 basis points.
Downward stress adjustment	$\text{downward stress} = \max(3\%\,; nominal\ risk\ free\ interest\ rate) \times\ (-0.20)$ The downward stress adjustment must be between 60 and 200 basis points.

The upward and downward stress adjustments must be added to the nominal risk-free interest rates or to the real yields if these are used explicitly in an asset valuation (APRA, 2023). The upward stress is larger than the downward stress, which was based on historical experience and an expectation that an upward movement is more likely than a downward movement of the same magnitude (APRA, 2013).

Equity stress

This module measures the impact on the capital base of a fall in the value of equity assets. Unlisted equities include assets not covered by other risk modules, such as precious metals, works of art, commodities or plant and equipment (APRA, 2023). The stress test for unlisted equities is higher than for listed equities because unlisted equities tend to have higher risk and less liquidity. In extreme conditions, the realisable value of unlisted equities may fall more than listed equities (APRA, 2011). Forward-looking volatility is a key valuation parameter for financial options and some other types of derivatives.

Table 2: Asset risk charge, equity stress

(APRA, 2023)

Listed equities	Increase the dividend yield on the ASX200 index at the reporting date by 2.5%. The same proportional fall in value (price) must be applied to both Australian and overseas listed equities. $fall\ in\ value = fair\ value\ \times \frac{(1-d)}{d'}$, or $stressed\ value = fair\ value\ \times \frac{d}{d'}$ where d is the actual dividend yield at the reporting date, and d' is the stressed dividend yield. The formula to determine the dividend yield is $dividend\ yield\ (\%) = \frac{annual\ dividends\ per\ share}{current\ share\ price} \times 100$ The ASX200 dividend yield is determined using dividends for the 12 months prior to the reporting date and asset values at the reporting date.
Unlisted equities and other assets	Increase the dividend yield on the ASX200 index at the reporting date by 3%. The same proportional fall in value (price) must be applied to all unlisted equities and assets. The ASX200 dividend yield is determined using dividends for the 12 months prior to the reporting date and asset values at the reporting date.
Volatility adjustment	Addition of 15% to the forward-looking equity volatility parameter for all durations.

The method of applying a stress to the dividend yield results in a higher capital charge being applied when the prevailing dividend yield is low, and a lower capital charge when the prevailing dividend yield is high (APRA, 2013). This reduces the pro-cyclical nature of the asset risk charge by increasing the charge following a rise in equity markets and reducing it after a market fall (APRA, 2013).

Property stress

This module applies to property and infrastructure assets. The property stress involves an addition to actual rental yields. Basing the asset stress on rental yields rather than prices reduces the pro-cyclical nature of the asset risk capital charge (APRA, 2023). This is similar to the equity module, where the stress is based on ASX200 dividend yields. The rental yields and changes in value can be calculated separately for each property asset, or they can be calculated for the property portfolio as a whole (APRA, 2023).

Table 3: Asset risk charge, property stress

Property assets	Increase the rental yield by 2.75%. Rental yields are based on the most recent leases in force, net of expenses. $fall\ in\ value = fair\ value\ \times \frac{(1-d)}{d'}$, or $stressed\ value = fair\ value\ \times \frac{d}{d'}$ where d is the actual rental yield at the reporting date, and d' is the stressed rental yield. The formula to determine the net rental yield is $net\ rental\ yield\ (\%) = \frac{annual\ rental\ income - annual\ expenses}{property\ value} \times 100$
Infrastructure assets	Increase the rental yield by 2.75%. Yields are rental yields before tax.

Currency stress

This module measures the impact of changes in foreign currency exchange rates. Life companies must calculate the impact on their capital base of both an appreciation and depreciation of the Australian dollar. In each scenario, the Australian dollar must be assumed to move in the same direction against all foreign currencies (APRA, 2023).

Table 4: Asset risk charge, currency stress

Appreciation of AUD	$stress = 25\%\ increase\ in\ the\ value\ of\ AUD\ against\ all\ foreign\ currencies$ $where\ stressed\ value = \frac{fair\ value}{1.25}$

Depreciation of AUD	$stress = 25\%\ decrease\ in\ the\ value\ of\ AUD\ against\ all\ foreign\ currencies$ $where\ stressed\ value = \frac{fair\ value}{0.75}$

Credit spreads stress

This module measures the impact of an increase in credit spreads and the risk of default. Credit spreads stress is applied to interest-bearing assets, including cash deposits, floating-rate assets, credit derivatives and zero-coupon instruments. A credit spread is the difference in yield between an asset that is subject to credit risk and a similar risk-free asset (APRA, 2013), for example Australian government bonds

Table 5: Asset risk charge, credit spreads stress

Credit spreads stress	The value of interest-bearing assets is reduced via specified increases in the prevailing yields on those assets. The stressed value of an asset is determined by increasing the current yield on the asset by a specified spread and then multiplying the reduced value of the asset by (1 – default factor). The credit spreads and default factors depend on the counterparty grade and the nature of the asset. The comprehensive credit spreads and default factors can be found in Table 1, paragraph 62 of LPS 114 (https://www.legislation.gov.au/F2023L00676/asmade/text)

Expected inflation stress

This module measures the impact of changes to expected CPI inflation rates (APRA, 2023). The stress does not apply to assets that are affected by equity or property stresses. An insurer's assets and liabilities whose values are dependent on expected inflation or nominal interest rates must be revalued using the stressed expected inflation and stressed nominal interest rates.

Table 6: Asset risk charge, expected inflation stress

Upward stress adjustment	$upward\ stress = 1.25\ \%$
Downward stress adjustment	$downward\ stress = \begin{cases} 0.50\ \% & i^* < 0\%\ p.a. \\ 0.50\ \% + 0.5(i^*) & 0\% < i^* < 1\%\ p.a. \\ 1.00\ \% & i^* > 1\%\ p.a. \end{cases}$ where i^* is the nominal risk-free interest rate and bp means basis points.

The upward and downward stress adjustments are added to the nominal risk-free interest rates and to any expected inflation rates included in asset valuations. A life insurer must calculate the impact of both an upward and downward stress adjustment on their capital base, and both calculations must be used in the aggregation formula (APRA, 2023).

Default

This module allows for the risk of default of counterparties to particular assets. The default factors for this stress are higher than for the credit spreads stress, which recognises that exposures to individual counterparties can be significantly greater for the default stress than for the credit spreads stress. APRA states in its March 2013 asset risk charge discussion paper that it expects bond portfolios to be well-diversified with each individual counterparty exposure being relatively small. The default factors by counterparty grade can be found in Table 2, paragraph 76 of LPS 114 (https://www.legislation.gov.au/F2023L00676/asmade/text)

4.5 Aggregation formula and correlation matrix

Under the Australian standard method, the risk components of the asset risk charge are combined according to an aggregation formula (APRA, 2023).

Equation 1 Australian asset risk charge aggregation formula

$$Asset\ Risk\ Charge = A_{default} + \sqrt{\sum_{x,y} Max\ (0, Corr_{x,y} \,.\, A_x \,.\, A_y \,.\, sign(x).\, sign(y))}$$

Where:

a) A_x is the risk charge component for asset risk stress x.

b) A_y is the risk charge component for asset risk stress y.

c) $\sum_{x,y}(\ldots)$ is the sum over all combinations of asset risk stresses, excluding the default stress.

d) $Corr_{x,y}$ is the correlation between asset risk stresses x and y.

e) $sign(x)$ is 1 for the equity, property and credit spreads stresses. For the real interest rates and expected inflation stresses, $sign(x)$ is 1 if the stress is a decrease in rates, otherwise it is -1. For the currency stress, $sign(x)$ is 1 if the stress is a depreciation of the Australian dollar against foreign currencies, otherwise it is -1; and

f) $sign(y)$ is defined in the same way as $sign(x)$.

And the correlation matrix is:

Table 7: Asset risk charge correlation matrix (APRA, 2023)

	RIR	INF	CUR	EQY	PROP	CSP
RIR	1.0	0.2	0.2	0.2	0.2	0.2
INF	0.2	1.0	0.2	0.4	0.4	0.2
CUR	0.2	0.2	1.0	0.6	0.2	0.4
EQY	0.2	0.4	0.6	1.0	0.4	0.8
PROP	0.2	0.4	0.2	0.4	1.0	0.4
CSP	0.2	0.2	0.4	0.8	0.4	1.0

The Australian Prudential Regulation Authority (APRA) outlined that it was challenging to confidently determining correlations between extreme events due to limited relevant historical data (APRA, 2013). As a result, APRA adopted a conservative approach in allowing recognition of diversification benefits.

In the aggregation formula, the correlations between the asset risk components are intended to capture the likelihood of the different stresses occurring simultaneously (APRA, 2013). The formula recognises that the probability of all the stresses occurring at the same time is very small. The real interest rates, expected inflation and currency stresses apply in two directions and have different signs for an upward or downward stress. If the signs of stresses x and y are opposite, their correlation is adjusted to zero and the total asset risk charge will be smaller than if the signs of the stresses were the same (APRA, 2013). Lower correlations also result in a lower aggregated asset risk charge.

The effect of the $sign(x)$ and $sign(y)$ terms can be illustrated with an example. A depreciation of the Australian dollar has a positive sign (+1), which is the same as the sign of the equity, property and credit spreads stresses. Therefore, a depreciation of the Australian dollar is positively correlated with these other stresses and the $Max\ (0, Corr_{x,y} . A_x . A_y . sign(x). sign(y))$ term will be non-zero. Conversely, an appreciation of the Australian dollar has a negative sign (-1). Negative correlations have not been allowed in order to limit the extent to which diversification benefits can be recognised (APRA, 2013), therefore an appreciation of the Australian Dollar is considered to be uncorrelated with these other stresses and the corresponding term $Max\ (0, Corr_{x,y} . A_x . A_y . sign(x). sign(y))$ in the aggregation formula will be zero.

For the real interest rates, expected inflation and currency stresses, normally only one direction of stress will produce a reduction in a life insurer's capital base. However, it is possible for the risk charge components to be non-zero in both directions if the assets or liabilities include financial derivatives or options, or if an insurer has assets and liabilities denominated in multiple foreign currencies (APRA, 2013). If a stress test produces non-zero risk charge components in both directions, it is possible that the smaller risk charge component could produce the larger aggregation. The aggregation formula must be calculated separately for each different combination of non-zero risk charge components. It is not correct to simply perform the aggregation once using the larger of the risk charge components. The occurrence of a non-negative risk charge in two directions is outside the scope of the modelling in this project, therefore this report does not include a numeric example. However, there is an example available on page 9 of APRA's March 2013 information paper which steps through the required approach for applying the aggregation formula when a risk charge component is non-negative in two directions. This is a useful resource if a more comprehensive explanation is desired.

The Australian Standard Method aggregates the 0.5 per cent marginal risk charges using a fixed correlation matrix based on Pearson correlation. This approach implicitly assumes an elliptical dependence structure, under which linear correlation adequately captures joint behaviour in the tails. However, Pearson correlation is only a natural dependence measure for elliptical distributions and does not capture tail dependence present in financial risks, meaning it may understate joint extreme losses and overstate diversification benefits in 1-in-200 stress scenarios (Embrechts et al, 1999).

5. Global approaches to capital requirements

5.1 European Union capital requirements: Solvency II

Solvency II is the regulatory framework that outlines capital requirements, risk management and reporting standards of insurance companies in the European Union (Leiser et al, 2023). A life insurer's capital requirement, the Solvency Capital Requirement (SCR) is set to achieve a 99.5% probability of remaining solvent over one year (Leiser et al, 2023), which is identical to the objective of the Australian Prudential Regulation Authority (APRA) solvency objective. Under Solvency II, the Basic Solvency Capital Requirement (BSCR) can be calculated using a standard formula, an internal model or a combination of the two. As illustrated in Figure 4, the BSCR is divided into risk categories, including three underwriting risk charges and two other risk charges. Each risk category is calibrated using a Value-at-Risk measure to achieve a 99.5% probability of solvency over one year (Leiser et al, 2023). The individual capital charges are then aggregated using an aggregation formula and predefined correlation matrices (Leiser et al, 2023).

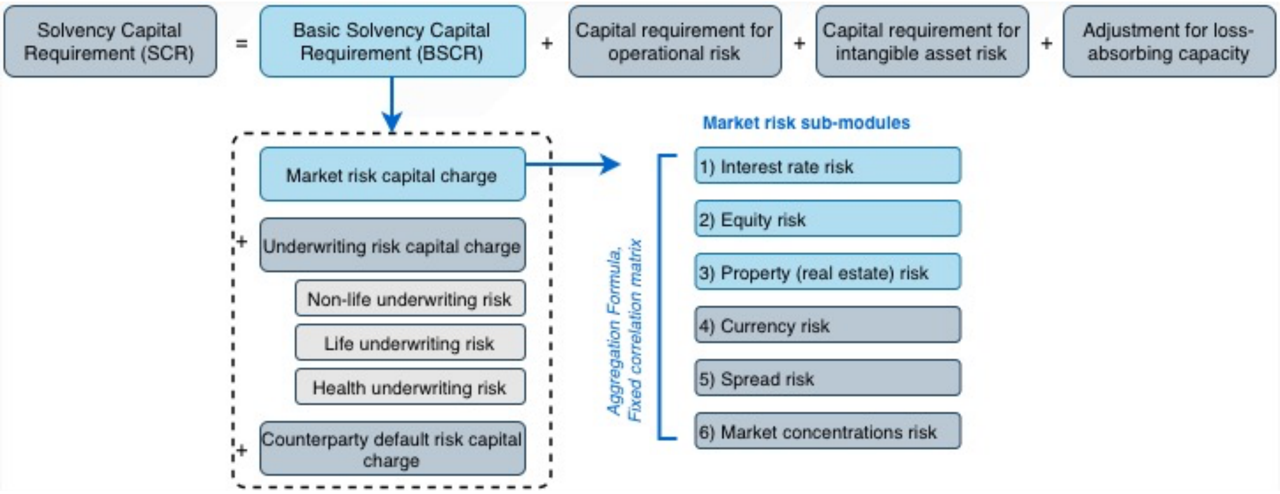

Figure 4: Overview of the Solvency II BSCR and market risk charge. (Created by author)

The Solvency II Basic SCR and the market risk sub modules are similar to the Australian PCA and asset risk charge. The market risk module seeks to capture the risk of a decrease in an insurer's surplus position. This can occur due to a fall in the value of assets held by an insurer, increases in the value of an insurer's non-insurance liabilities, or if both assets and liability values fall or rise simultaneously.

The risk of a decrease in surplus position is captured through a series of sub-modules addressing different market factors that may affect the value of assets and liabilities. In each case, a risk charge is calculated that contributes to the overall capital charge for market risk, which directly mirrors the Australian approach.

The overall capital requirement for market risk is calculated using an aggregation formula and correlation matrix set out in the Level 2 Delegated Regulation (European Parliament, C. o. t. E. U., 2009).

Equation 2 The Solvency II SCR aggregation formula

$$SCR_{market} = \sqrt{\sum_{i,j} Corr_{i,j} \times SCR_i \times SCR_j},$$

Within the SCR formula, SCR_i denotes sub-module i and SCR_j denotes sub-module j (Leiser et al, 2023).

The correlation factors between the market risk sub modules are (European Parliament & Council of the European Union, 2009):

Table 8: Solvency II market risk correlation matrix

	Interest rate	Equity	Property	Spread	Concentration	Currency
Interest rate	1.00	A	A	A	0.00	0.25
Equity	A	1.00	0.75	0.75	0.00	0.25
Property	A	0.75	1.00	0.50	0.00	0.25
Spread	A	0.75	0.50	1.00	0.00	0.25
Concentration	0.00	0.00	0.00	0.00	1.00	0.00
Currency	0.25	0.25	0.25	0.25	0.00	1.00

Where "A" denotes a value of 0.5 when the market is up, and 0 if down.

A key distinction between Solvency II and the Australian Standard Method lies in how diversification is recognised in the aggregation formula. Under Solvency II, the term $Corr_{i,j} \times SCR_i \times SCR_j$ for pairs of market-risk sub-modules with "A" correlations (interest rates with equity, property, or spread risk) becomes zero when the market is down because the stressed capital components do not change sign. In contrast, the Australian aggregation formula explicitly incorporates the sign of each stress. Under the Australian Prudential Regulation Authority (APRA) approach, the term $Max\ (0, Corr_{x,y} \,.\, A_x \,.\, A_y \,.\, sign(x).\, sign(y))$ only reduces to zero when the signs of the risk-charge components differ.

The stresses used to calculate each Solvency II market-risk capital charge are broadly similar to those under the Australian regime, although Solvency II uses fixed parameters, whereas APRA's stresses vary with market conditions (e.g., rental yields, dividend yields). Solvency II applies a 39% fall to listed equities and a 49% fall to all other equities; property assets are shocked by an immediate 25% fall (Leiser et al., 2023). Currency risk is stressed by a 25% exchange-rate movement (Leiser et al., 2023), consistent with the Australian standard. The complete set of market-risk stresses is defined in Articles 100–105 of Directive 2009/138/EC, but further detail is out of scope for this thesis.

The Solvency II framework has attracted considerable academic scrutiny. Wiehenkamp (2010) characterises it as a shift from formula-based capital rules to a more comprehensive risk-based system. However, several authors highlight material limitations. McCullough (2014) argues that the equity risk module may not reflect actual risk due to backward-looking calibration and basis risk, while Mittnik (2016) contends that the market-risk calibrations are fundamentally flawed and may produce unstable capital requirements. These limitations are equally applicable to the Australian Prudential Regulation Authority (APRA) capital requirements.

Although internal models are often more reflective of a firm's true risk profile (McCullough, 2014), their adoption is challenging. Approval requires extensive documentation (Field, 2015) and meeting six standards - Use Test, Statistical Quality, Calibration, Profit and Loss Attribution, Validation, and Documentation (Austin et al., 2009). These standards demand evidence of actual business use, highly granular data systems, and robust statistical justification. A further challenge is the reliance on expert judgement when specifying correlations between risks. Austin et al. (2009) emphasise that correlations are unlikely to remain stable over time, making them difficult to estimate and justify. They argue that although individual risks can be modelled with reasonable confidence, Solvency II aggregates them using correlation assumptions that firms and regulators often cannot comprehensively substantiate, which echoes APRA's disclosure of applying substantial judgement when deriving the stresses applied to each risk charge component and the correlation values in the aggregation formula.

5.2 Canadian solvency requirements: LICAT

In Canada, required capital for life insurers is calculated using a Life Insurer Capital Adequacy Test (LICAT), which is a model-based projection of future cash flows. LICAT permits greater actuarial judgement than formula-based regimes but requires regulatory review and approval of key assumptions and methodologies (Leiser et al, 2023).

The minimum required capital, known as the Base Solvency Buffer (BSB), is calibrated to approximate a 99% Conditional Tail Expectation (CTE) over a one-year horizon (Leiser et al, 2023). CTE represents the probability-weighted average loss given that the loss exceeds a defined threshold.

The 99 per cent CTE calibration underlying the BSB focuses on the average severity of losses in the extreme tail once a stress threshold has been breached, whereas the Australian Prudential Regulation Authority (APRA) 0.5% Value at Risk corresponds to a single quantile defining a 99.5 per cent probability of sufficiency. As a result, CTE is generally more conservative than Value at Risk for the same confidence level, as it captures both the likelihood and magnitude of extreme losses rather than only the threshold loss (Kirk, 2025).

The BSB is the sum of the capital requirements for five risk components, as illustrated in Figure 5.

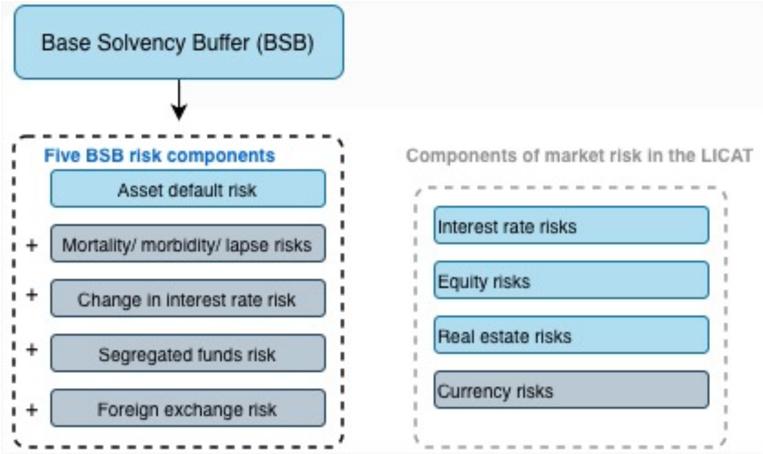

Figure 5: Overview of the Canadian BSB and market risk components. (Created by author)

The market-risk elements most comparable to the Australian PCA are (Leiser et al, 2023):

- Asset default risk, which captures losses in the market value of equities, property, and other assets due to credit deterioration or default.
- Change in interest rate risk, which reflects losses arising from movements in the interest-rate environment.
- Foreign exchange risk, which captures losses from adverse currency-rate movements.

Market risk is also captured through the segregated fund risk component. Here, the insurer is exposed if the segregated fund's assets underperform the guaranteed benefit, requiring the insurer to fund any shortfall (Leiser et al, 2023). This element is specific to the structure of Canadian Life Assurance.

While the LICAT framework contains some prescribed correlations, its structure differs from both Solvency II and the Australian Standard Method. LICAT does not apply fixed correlations within the market-risk modules themselves (Navratil, 2024). The credit risk and market risk capital requirements are added together to form a single combined amount, with no diversification or correlation applied at this step (OSFI, 2025). This combined amount is then aggregated with the insurance risk requirement using an assumed 50% correlation between the two classes to produce a diversified requirement (OSFI, 2025).

5.3 US capital requirements: NAIC RBC

In the United States, the National Association of Insurance Commissioners (NAIC) sets Risk-Based Capital (RBC) requirements, which are primarily formulaic and factor-based, although certain market-risk components use model-based calculations. RBC forms part of the broader solvency-monitoring framework but does not specify a minimum capital requirement. Instead, it serves as an objective trigger for regulatory intervention and is designed to identify weakly capitalised insurers through a simple, cycle-neutral structure that relies on publicly available data.

RBC establishes a reference-point level of required capital that regulators compare with an insurer's statutory capital (Leiser et al, 2023). Unlike the Australian Standard Method or Solvency II, the RBC calculation is not calibrated to a specific probability level (e.g. 99.5% sufficiency). In practice, the combination of statutory reserves and minimum capital requirements is generally expected to provide around 95% protection over a five- to seven-year horizon (Leiser et al, 2023).

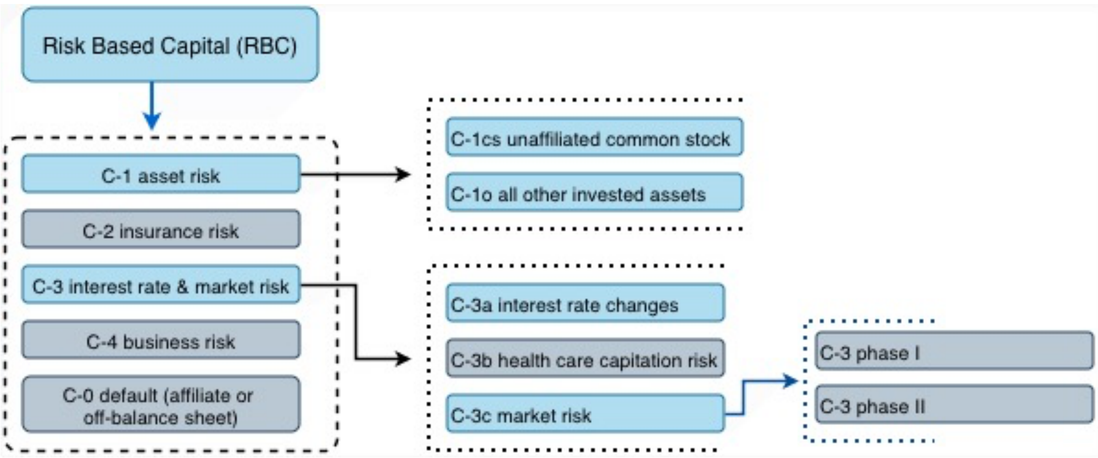

Figure 6: Overview of the US RBC framework and market risk components. (Created by author)

The Life RBC (LRBC) system consists of separate components for major risk categories (C-0 through C-4), which are aggregated to determine total required capital as outlined in Figure 6. Market-related risks fall mainly within:

- C-1 (Asset risk), which captures credit loss or non-performance of investments, including equity and other asset value declines.
- C-3 (Interest rate and market risk), which reflects changes in asset values and policy cash flows due to market movements. For long-dated guarantees such as variable

annuities or single-premium life insurance, C-3 calculations require a model-based approach.

The LRBC system assumes that statutory policy reserves already cover expected losses that occur at one standard deviation, and LRBC provides a cushion for risk levels and losses that occur under more adverse conditions (Leiser et al, 2023). The key solvency indicator is a ratio, with regulatory action triggered when this ratio falls below defined thresholds.

Equation 3 US key solvency indicator ratio

$$solvency\ indicator\ ratio = \frac{available\ capital}{required\ capital},$$

The calculation of the assessment capital requirement (ACR) is as according to the ACR formula.

Equation 4 US Assessment Capital Requirement (ACR) formula

$$ACR = C0 + C4a + \sqrt{(C1o + C3a)^2 + (C1cs + C3c)^2 + C2^2 + C3b^2 + C4b^2}$$

Each term corresponds to a specific risk category: affiliate asset risk ($C0$), business risk ($C4a$), other invested asset risk ($C1o$), interest rate risk ($C3a$), common stock asset risk ($C1cs$), market risk ($C3c$), insurance risk ($C2$), health capitation risk ($C3b$), and health administration expense risk ($C4b$).

As seen from the formula, fixed correlations are not currently used in LRBC calculations (Navratil, 2024). However, the American Academy of Actuaries' Life RBC Working Group initiated a 2024 review of potential correlation structures, including those in Solvency II, signalling possible future developments in the aggregation approach.

5.4 Jurisdiction summary

Table 9: Summary of capital requirements for market risk between jurisdictions

	Australia	European Union	Canada	United States
Use of correlation matrices	✓	✓		
Target confidence level/ probability of sufficiency	99.5% over 1 year	99.5% over 1 year	99% CTE over 1 year	Not clearly specified
Primary methodology for market/ asset risk	Stress-based risk charges by asset risk component	Stress-based capital by market risk sub-module	Stress and factor based, aggregated additively	Factor based (C-1) and scenario-based interest and market risk (C-3)
Diversification recognition at which level?	Within asset risk charge (correlations) and across PCA components (correlations)	Within market risk (correlations) and across BSCR modules (correlations)	No diversification within market risk	No diversification within C-1 or C-3.
Aggregation formula structure	Square root correlation for asset risk charge module and PCA	Square root correlation for market risk module and BSCR	Additive within market risk, square root formula only across asset and insurance risk	Additive within C components

6. Copulas

A copula is a function that links the joint distribution of multiple random variables to their marginal distributions, allowing the dependence structure to be modelled separately from the behaviour of each individual variable. This separation is particularly useful because defining a full joint distribution becomes increasingly difficult as the number of variables grows and as relationships become non-linear or asymmetric. Each joint distribution embeds its own dependence structure, and copulas provide a flexible way to model this dependence without imposing restrictive assumptions on the marginals.

In the context of market-risk modelling, copulas offer a powerful alternative to traditional linear correlation measures by allowing more complex interactions between variables, including non-linear dependence and tail dependence. This is especially important for capturing the joint behaviour of asset classes during stressed market conditions.

For this thesis, the focus is on the simultaneous behaviour of equity, property and fixed-interest assets on a life insurer's balance sheet. A copula approach enables a more risk-sensitive representation of how these asset classes interact, particularly in the lower tail, which is critical for assessing regulatory capital requirements. The motivation for using copulas is that it is generally easier and more realistic to model individual marginal distributions and then combine them through an appropriate dependence structure, rather than attempting to specify an entire multivariate distribution directly.

6.1 Basic theory

Before the development of copulas, multivariate models were typically constructed by extending univariate distributions to higher dimensions. This approach had several limitations:

1. it required all marginal distributions to belong to the same family (e.g. multivariate normal or t), which is often unrealistic when variables follow different distributions,
2. the mathematical complexity of extending these families beyond two dimensions made joint densities difficult or impossible to express in closed form, and
3. dependence parameters were embedded within the marginal distributions, preventing a clear separation between modelling individual behaviour and modelling dependence.

Copula theory overcomes these issues by allowing variables with different marginal distributions to be combined under a common dependence structure. The copula is specified independently of the marginals (Genest & Favre, 2007), which provides significantly greater flexibility and interpretability in multivariate modelling.

Sklar's theorem states that any multivariate joint distribution can be expressed using its marginal distributions and a copula (Nelsen, 2006). The theorem is especially useful for multivariate modelling because the marginal distributions and the copula do not need to belong to the same family of distributions (Fan, 2014). In the bivariate case, let $F_{XY}(x, y)$ be a joint CDF with marginal distributions $F_X(x)$ and $F_Y(y)$. Then there exists a copula C such that for all $x, y \in [-\infty, \infty]$: $F_{XY}(x, y) = C[F_X(x), F_Y(y)]$. If $F_X(x)$ and $F_Y(y)$ are continuous, then the copula C is unique. Conversely, if C is a copula and $F_X(x)$ and $F_Y(y)$ are univariate CDFs, then $F_{XY}(x, y)$ is a joint CDF with marginal distributions $F_X(x)$ and $F_Y(y)$ (Nelsen, 2006).

Understanding copulas also requires a clear understanding of dependence measures, particularly those relevant to financial tail events.

6.1.1 Measures of dependence

Measures of dependence are tools to explain a complex dependence structure (Schmidt, 2007). Correlation is a common way to describe dependencies, for example Pearson's correlation coefficient, Kendall's tau and Spearman's rho.

Pearson correlation assumes a linear relationship, and measures only the strength and direction of a linear association between two variables. Kendall's tau (τ) and spearman's rho (ρ_S) are rank-based alternatives which better capture non-linear dependencies. They do not assume a linear functional form and are less sensitive to extreme values, which is crucial for examining the often-volatile nature of financial returns. Kendall's tau measures the similarity of the orderings of the data when ranked by each of the quantiles (Kendall, 1938). The Spearman correlation between two variables is the Pearson correlation between the rank values of those two variables (Spearman, 1904). Spearman's rank correlation coefficient assesses monotonic relationships, whether linear or not.

Another measure of dependence is tail dependence, which is concerned with the extreme values of a distribution (Schmidt, 2007).

Traditional methods for measuring dependence between assets have significant limitations in capturing the complexity of financial market relationships. Financial asset returns often exhibit non-linear, asymmetric and tail-dependent behaviours, especially during periods of market shocks (Embrechts et al, 1999) and financial markets tend to be more correlated when markets fail (Sherris et al, 2008). A copula approach can better capture non-linear dependencies and tail dependence to provide a more accurate and nuanced representation of the co-movement of different assets under various market conditions. Different copulas have different dependence structures, and vastly different tail dependence. The appropriate copula family and parameters can be selected to suit the specific financial market behaviour of interest, for example extreme positive or negative co-movements.

6.1.2 Tail dependence

Since financial markets have a higher correlation when markets fail (Sherris et al, 2008), the dependence characteristics in the tails are of particular importance in the determination of regulatory capital requirements.

For a bivariate copula $C(u, v)$, tail dependence measures how strongly two variables tend to experience extreme outcomes together. The upper tail dependence coefficient, λ_U, measures the probability that Y is extreme (above a high quantile), given that X is extreme. A value of $\lambda_U = 0$ indicates asymptotic independence in the upper tail, while $\lambda_U > 0$ implies tail dependence. The lower tail dependence coefficient, λ_L, captures the probability that both variables jointly experience extreme low outcomes. A value of $\lambda_L = 0$ indicates asymptotic independence in the lower tail, while $\lambda_L > 0$ implies tail dependence.

In the 3-dimensional case, tail dependence measures the probability that multiple variables are simultaneously extreme. The upper tail dependence coefficient, $\lambda_U^{(3)}$, measures the conditional probability that at least two or all three variables take on extreme high values given that one does. Similarly, the lower tail dependence coefficient, $\lambda_L^{(3)}$, is the joint CDF of the three uniforms simultaneously experience extreme low outcomes.

6.2 Copula types and families

Two types of copula groups are the Elliptical and Archimedean family. Elliptical copulas correspond to an elliptical distribution by Sklar's theorem (Yan, 2006) and are particularly used to describe dependencies when relationships are approximately linear or symmetric.

Archimedean copulas can model non-linear and asymmetric tail dependence and are defined by a generator function which determines the dependence structure (Nelson, 2006). The Archimedean copula family has wide applications due to the ease of construction, the variety copulas within the family, and their properties (Nelson, 2006). Archimedean copulas are used to correlate a potentially large number of similar variables and require only a single parameter, θ, which controls the degree of the spread. In their standard form, Archimedean copulas can model only positive correlations; however, "reflected" versions of these copulas also exist (Nelson, 2006).

The following sections will introduce and explore five specific types of copulas with unique capabilities to capture different aspects of dependence.

6.3 Elliptical copula family

6.3.1 Gaussian copula

The Gaussian (Normal) copula is popular in financial modelling due to its simplicity and mathematical tractability. It is derived from the multivariate normal distribution. For two uniform random variables U and V, a 2-dimensional Gaussian copula with correlation parameter θ is defined as (Nelsen, 2006) $C_{Gaussian}[u, v;\ \theta] = \Phi_{\theta}[\Phi^{-1}(u), \Phi^{-1}(v)]$, where Φ^{-1} is the inverse of the cumulative distribution function (CDF) of the univariate standard normal distribution, and Φ_{θ} is the bivariate standard normal CDF with correlation θ.

The Gaussian copula can be extended beyond two dimensions (Yan, 2006). Consider the joint CDF $\Phi_{\Sigma,d}$ of a multivariate d-dimensional normal distribution with correlation matrix Σ, where Φ is the CDF of a standard normal variable. A d-dimensional Gaussian copula with dispersion matrix Σ has the form $C[u_1, \dots, u_d;\ \sum] = \phi_{\Sigma}[\phi^{-1}(u_1), \dots, \phi^{-1}(u_d)]$. The Gaussian copula has no tail dependence, $\lambda_L = \lambda_U = 0$, therefore it is most appropriate when the relationship between variables is primarily linear, especially when extreme co-movements (tail dependence) are not a primary concern. Therefore, the Gaussian copula is consistent with the dependence measured by Pearson correlation, since its dependence structure is fully characterised by linear correlation under an implicit elliptical distributional assumption.

This copula serves as a baseline model to compare against more flexible copulas that are able to capture non-linear and tail dependencies.

6.3.2 Student's t copula

The Student's t copula extends the Gaussian copula by incorporating degrees of freedom, ν, allowing it to model tail dependency and capture more extreme co-movements between variables. The degrees of freedom parameter ν controls the strength of tail dependence (Yan, 2006). Smaller values of ν correspond to heavier tails and a higher probability of joint extreme outcomes. As the degrees of freedom parameter ν increases, the Student's t copula approaches the Gaussian copula.

The bivariate Student's t copula has the form $C_t[u, v;\ \theta, v] = \mathrm{t}_{\theta,v}[{t_v}^{-1}(u), {t_v}^{-1}(v)]$. In equation $C_t[u, v;\ \theta, v]$, θ is the correlation coefficient between the two variables, w ($w > 0$) is the degrees of freedom parameter, ${t_w}^{-1}$ is the inverse CDF of the univariate Student's t distribution with w degrees of freedom, and $\mathrm{t}_{\theta,w}$ is the bivariate Student's t CDF with correlation θ and w degrees of freedom (Embrechts et al, 1999).

The Student's t copula can also be extended to d-dimensions (Yan, 2006). Consider the joint CDF $\mathrm{T}_{\sum,w}$ of the standardized multivariate Student's t distribution with correlation matrix $\sum$ and w degrees of freedom. Let F_{t_w} be the CDF of the univariate t distribution with w degrees of freedom. Then, a Student's t copula with dispersion matrix $\sum$ and degrees-of-freedom parameter w is defined as $C[u_1, \dots, u_d;\ \sum] = \mathrm{T}_w\ [{F_{t_w}}^{-1}(u_1), \dots, {F_{t_w}}^{-1}(u_d)]$. The Student's t copula has symmetric tail dependence, $\lambda_L = \lambda_U > 0$, and is best suited for financial applications where extreme co-movements, both positive and negative, are of interest. It provides a more realistic modelling of risk than the Gaussian copula by accounting for tail dependencies.

Venter et al (2007) and Venter (2003) outline two major limitations of the Student's t copula and the implications of these. These limitations are the symmetry between right and left tails, and the single degrees-of-freedom parameter. In insurance claims costs or losses, stronger tail dependence tends to happen in the right tail. Putting the same dependence in the left tail would be inaccurate, however in an insurance context it would likely have little effect on risk measurement overall, which is largely affected by the right tail (Venter et al, 2007).

In studies examining different copulas for modelling financial market dependencies and tail risks across asset classes, the t-copula consistently outperformed other copulas in capturing the dependence structure of portfolio returns, particularly for joint extreme downward movements (Kole et al., 2005; Koedijk et al., 2006; Necula, 2010).

6.4 Archimedean copula family

An Archimedean copula with a strict generator function has the form $C[u_1, \dots, u_d] = \phi^{-1}\{\phi(u_1) + \cdots + \phi(u_d)\}$ (Ruppert, 2011). The generator function determines the strength of the dependence between variables. Simpler forms of the generator lead to easier calculations for tasks like maximum likelihood estimation. Many Archimedean copulas are defined by a single parameter embedded within the generator function, simplifying the modelling process.

6.4.1 Clayton copula

The Clayton copula generator function is $\varphi(t) = \frac{1}{\theta}\left(t^{-\theta} - 1\right) = \frac{(t^{-\theta}-1)}{\theta}$, where $\theta > 0$ is the dependence parameter (Nelsen, 2006). The bivariate Clayton copula has the form $C_{Clayton}[u, v;\ \theta] = (u^{-\theta} +\ v^{-\theta} - 1)^{-\frac{1}{\theta}}$, which can be extended to d-dimensions for any $d \geq 2$ and $\theta > 0$ to $C[u_1, \dots, u_d;\ \theta] =\ ({u_1}^{-\theta} + \cdots +\ {u_d}^{-\theta} - d + 1)^{-\frac{1}{\theta}}$ (Nelsen, 2006).

The Clayton copula exhibits tail dependence in the lower tail, $\lambda_L > 0$ and $\lambda_U = 0$, and exhibits monotonic dependence. The dependence increases as the dependence parameter θ increases. The Clayton copula is appropriate when the joint occurrence of extreme negative events is the primary concern. For example, simultaneous negative returns of financial assets, which is has applications in insurance, risk management and hedging strategies.

6.4.2 Gumbel copula

The Gumbel copula generator function is $\varphi(t) = (-\ln t\,)^{\theta}$, where $\theta \geq 1$ is the dependence parameter (Nelsen, 2006). The bivariate Gumbel copula has the form $C_{Gumbel}[u, v;\ \theta] =\ e^{-[(-\ln u)^{\theta} + (-\ln v)^{\theta}]^{\frac{1}{\theta}}}$, which can be generalised to d-dimensions for any $d \geq 2$ and $\theta \geq 1$ in the form $C[u_1, \dots, u_d] =\ e^{-[(-\ln u_1)^{\theta} + \cdots + (-\ln u_d)^{\theta}]^{\frac{1}{\theta}}}$.

The Gumbel copula exhibits tail dependence in the upper tail; $\lambda_U > 0$ and $\lambda_L = 0$, and also exhibits monotonic dependence. It is well-suited to assess the likelihood of simultaneous extreme gains. For example, the co-occurrence of asset price surges.

6.4.3 Frank copula

The Frank copula can to model dependencies without exhibiting tail dependence. While both the Gaussian and Frank copula are symmetric and exhibit no tail dependence, the Frank copula captures the strongest dependence in the centre of the joint distribution. The dependence parameter is related to Kendall's tau. This contrasts with the Gaussian linear dependence structure, where the dependence measure is driven by Pearson correlation.

The Frank copula generator function is $\varphi(t) = -\ln(\frac{e^{-\theta t}-1}{e^{-\theta}-1})$, where $\theta \neq 0$ is the dependence parameter (Nelsen, 2006). The bivariate Frank copula has the form $C_{Frank}[u, v;\ \theta] = -\frac{1}{\theta}\ln(1 + \frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{(e^{-\theta}-1)}$, and can be generalized to d-dimensions for any $d \geq 2$ in the form $C[u_1, \dots, u_d] = -\frac{1}{\theta}\ln(1 + \frac{(e^{-\theta u_1}-1)\dots(e^{-\theta u_d}-1)}{(e^{-\theta}-1)^{d-1}}$.

The Frank copula has no tail dependence, $\lambda_L = \lambda_U = 0$. It also exhibits bounded dependence, modelling increasing central dependence as θ increases. The Frank copula is a complementary model to the Clayton and Gumbel copulas by providing flexibility in capturing dependencies that are not in the extreme tails, but still significant.

6.5 Copula dependence structures

Figures 7 and 8 illustrate the varying dependence structures of common bivariate copulas. It is clear that the Clayton copula exhibits stronger dependence in the lower left region, the Frank copula has symmetric heavier dependence in the centre, and the Gumbel copula exhibits stronger dependence in the upper right region (Venter, 2002). The Student's t copula is illustrated with both 3 and 30 degrees of freedom to demonstrate the effect of varying the degrees of freedom parameter, w. As seen in Figure 7, lower values of w lead to heavier tails and stronger tail dependence (Jan, 2006), and as the degrees of freedom increase, the contour plot more closely resembles the Gaussian copula (Jan, 2006).

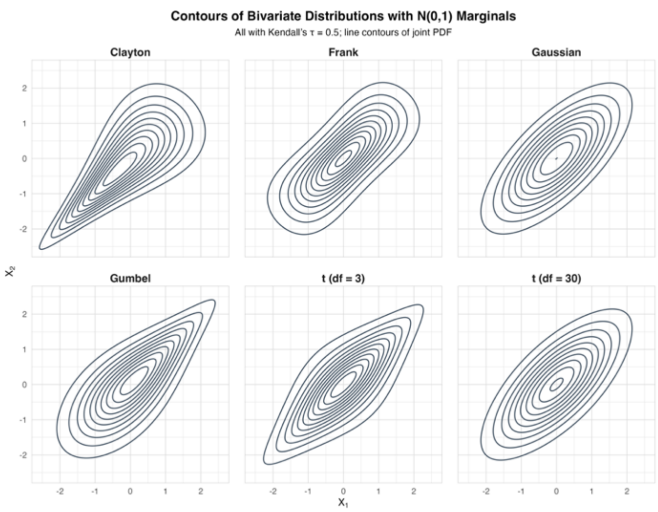

Figure 8 Contour plots of bivariate copulas with standard normal marginals. (Created by author)

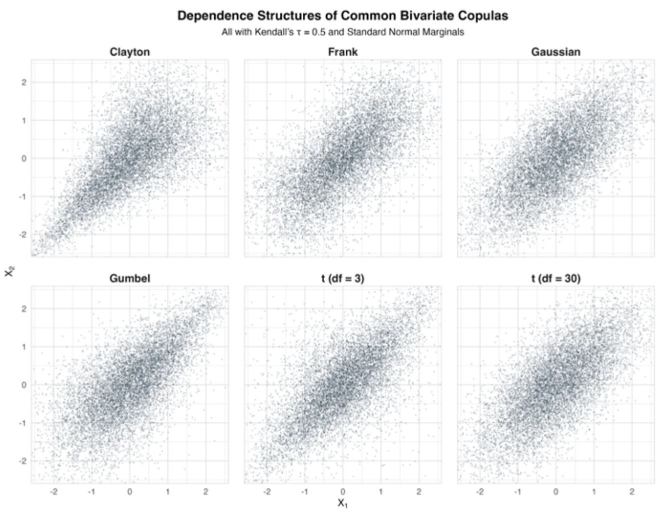

Figure 7 10,000 simulated points from bivariate copulas with standard normal marginals. (Created by author)

7. Methodology

7.1 Isolating market risk and constructing a notional balance sheet

As outlined in section 4.3, this thesis will investigate the Australian asset risk charge to isolate regulatory capital requirements for market risk. Under the standard method, the capital charge for each risk component within the asset risk charge is the fall in the insurer's capital base relative to the unstressed position.

To investigate both a copula-based and the standard approach, I first constructed a simplified notional balance sheet to represent a typical Australian life insurer. The balance sheet consists of assets, liabilities, and the resulting shareholders' equity or surplus value. My focus is on the capital requirements for market risk; therefore the types of assets and their joint behaviour are of primary concern.

My notional balance sheet is based on the annual reports of significant life insurers in Australia. In LPS 001 Definitions (APRA, 2015), the Australian Prudential Regulation Authority (APRA) considers life companies with total assets above AUD10 billion as significant financial institutions (SFIs). As of 30 June 2025, there are three SFI life insurers; Challenger Life Company Limited, Resolution Life Australasia Limited, and TAL Life Limited (APRA, 2025).

Challenger Limited is ASX-listed with two core businesses, Life and Funds Management. Challenger Life Company Limited is Australia's largest provider of annuities, managing around AUD25 billion in assets as of 31 March 2025 (Challenger, 2025). While Challenger's funds management business operates internationally, with offices in Australia, the UK, Europe and Asia, its life insurance and annuity business is focused on Australian residents (Challenger, 2025). Challenger's annual reports also clearly differentiate between its assets supporting the Australian life insurance operations, and those relating to its fund management business, making it suitable to inform the construction of a simplified notional balance sheet for the purposes of this report.

Conversely, Resolution Life Australasia and TAL Life Limited are both subsidiaries of large global parent companies. Resolution was acquired by Nippon Life Insurance Company on 30 October 2025. Following the acquisition, Resolution Life Australasia was merged with MLC Life Insurance and now operates under the Acenda brand. The Australian life insurance operations are a small portion of the overall operations, and the published balance sheet is heavily distorted by the international and non-life parts of the business.

TAL Life Limited and its Japanese parent company, Dai-ichi Life Holdings, Inc., do not publish regular or comprehensive annual reports, which further impeded using it as a basis for my notional balance sheet.

Therefore, the 2023, 2024 and 2025 Challenger Life annual reports were used to inform a notional balance sheet representative of a typical Australian insurer. Three asset risk charge components could be clearly linked to assets in Challenger Life's balance sheet. These components were real interest rates (RIR), equity (EQY) and property (PROP), and are represented by fixed income, equity and property assets respectively.

The values obtained from the annual reports are summarised below (Challenger, 2025):

Table 10: Challenger Life balance sheet breakdown

	Challenger Life 2025 (AUD '000)		Challenger Life 2024 (AUD '000)		Challenger Life 2023 (AUD '000)		Average (AUD '000)	
Total Assets	27,249,000	100%	26,528,600	100%	25,573,500	100%	26,450,366	100%
Equity	603,400	2.21%	448,300	1.69%	291,300	1.14%	447,666	1.68%
Property	2,733,100	10.03%	2,761,600	10.41%	3,062,400	11.97%	2,852,366	10.80%
Fixed Income	18,847,600	69.17%	18,333,500	69.11%	17,799,600	69.60%	18,326,900	69.29%
Other Assets	5,064,900	18.59%	4,985,200	18.79%	4,420,200	17.28%	4,823,433	18.22%
Total Liabilities	23,384,500		22,643,400		21,409,100		22,479,000	
Shareholders' Equity	3,864,500		3,885,200		4,164,400		3,971,366	

The notional balance sheet was constructed using a rebalancing factor such that the shareholders' equity (the "surplus") at time 0 was equal to 100. This was done purely to aid in discussions and to simplify the interpretability of results.

Each component of the notional balance sheet was multiplied by the rebalancing factor to obtain the final values, where $rebalancing\ factor = \frac{100}{shareholders'\ equity}$. Therefore, it follows that $notional\ total\ assets = average\ total\ assets\ \times\ rebalancing\ factor$, and the other components of the balance sheet can be derived in the same manner. Additionally, $surplus = total\ assets - total\ liabilities$.

Table 11: Notional balance sheet at t=0, based on Challenger Life balance sheets (2025, 2024, 2023)

	Rounded to 4dp	
Total Assets	667.3401	100.00%
Equity	11.3825	1.68%
Property	71.7803	10.80%
Fixed Income	462.3382	69.29%
Other Assets	121.8391	18.22%
Total Liabilities	567.3401	
Surplus	100	

The number of interest is the surplus value after 1 year. This surplus value equals total assets minus total liabilities. To isolate the impact of market risk, all elements of the balance sheet - except equity, property and fixed income assets - were assumed to remain constant. This assumption is of limited importance for equity and property in traditional life assurance products but is more material for interest rates due to the impact on the discount factor.

7.2 Copula modelling workflow

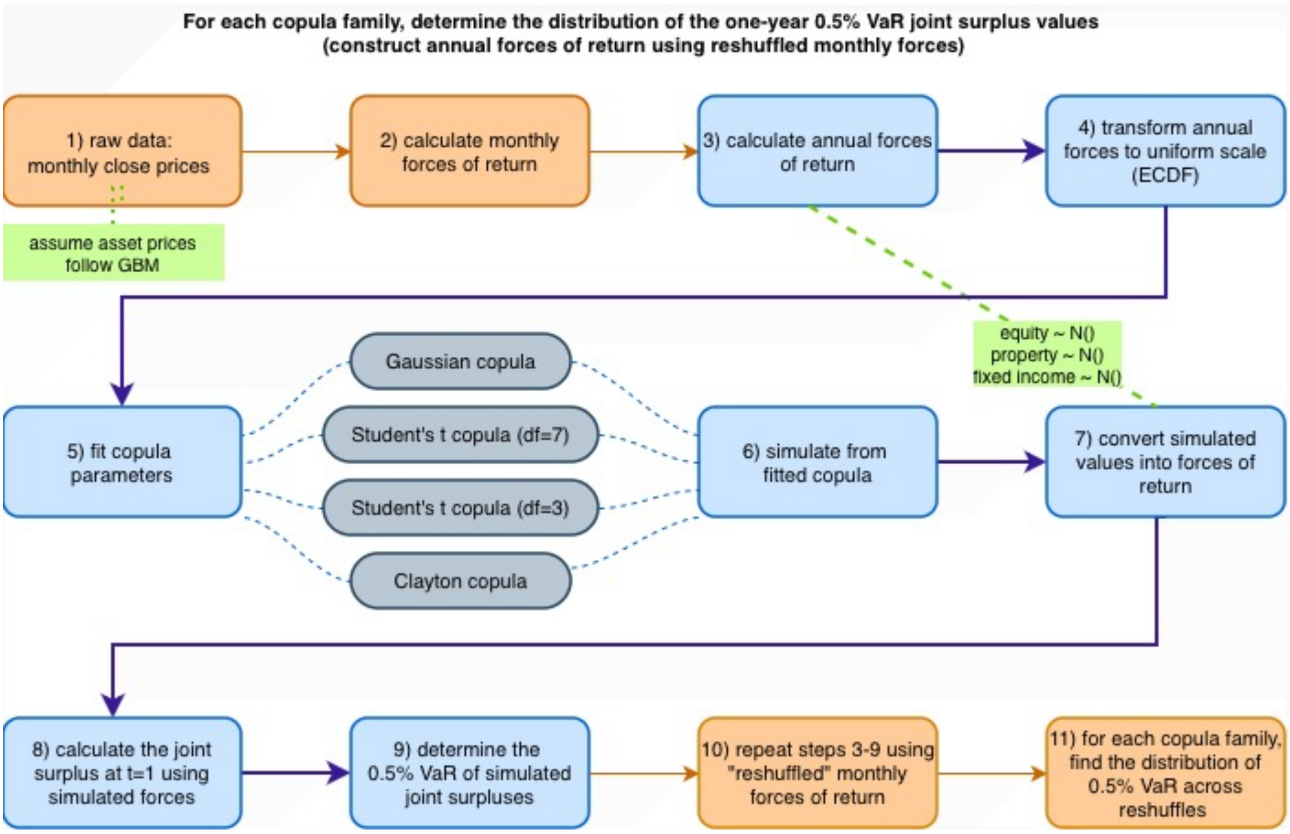

Figure 9: Copula modelling workflow. (Created by author)

7.2.1 Input data

The analysis used historical monthly asset prices obtained from LSEG Workspace, covering the period 30 June 2005 to 30 June 2025. Three asset classes were considered: equity, property and fixed income. Each asset class was represented by a listed market proxy. These are summarised in Table 13.

Table 12: Listed proxies for equity, property and fixed income assets

Equity	S&P/ ASX 200 accumulated index (AXJOA)
Property	S&P/ASX 200 A-REIT accumulation index (AXPJA)
Fixed income	Vanguard Australia fixed interest index fund (30 June 2004 – 30 July 2015) S&P/ ASX Australia aggregate bond index (31 August 2015 – 30 June 2025)

The S&P ASX 200 Accumulation Index AXJOA was selected as the equity proxy because it measures the total return received by investors. Unlike the corresponding price only index, AXJOA reinvests all cash dividends at the ex-dividend date and therefore provides a more realistic and comprehensive representation of long-term equity performance.

The S&P ASX 200 A REIT Accumulation Index AXPJA was selected as the property proxy. Distributions represent a material component of A REIT total returns, so the use of an accumulation index is necessary to accurately capture overall performance rather than price movements alone. Notably, there is a cross-contamination of equity systematic risk because these are listed companies.

For fixed income, complete index history was not available on LSEG Workspace. The S&P ASX Australia Aggregate Bond Index is available only from 31 August 2015. To extend the fixed income time series back to 2005, monthly net asset value data from the Vanguard Australian Fixed Interest Index Fund was used for the period 30 June 2004 to 30 July 2015. This fund seeks to track the Bloomberg AusBond Composite Zero Plus Year Index, which is the long-established benchmark for the Australian investment grade bond market (APRA, Undated). Monthly changes in net asset value provide a reasonable proxy for monthly price movements, acknowledging that the data reflects the unit trust management fees. A comparison between the monthly forces of return from the Vanguard fund and the S&P ASX Australia Aggregate Bond Index is presented in Figure 10 to illustrate the consistency of the two series over their overlapping period.

The analysis initially relied on annual financial year end price observations. However, monthly price data was adopted in order to increase the number of observations available for estimating marginal distributions, to strengthen the dependence modelling and to support the reshuffling method used later to construct pseudo annual forces of return.

Index proxies were used rather than investable funds because the aim of the analysis is to model systematic market behaviour and not manager specific performance. Indices provide representative exposure to each asset class and are standard in solvency, capital modelling and asset liability applications.

7.2.2 and 7.2.3 Calculate forces of return

For each asset series the raw input data consisted of monthly closing prices or net asset values. These price series were converted to monthly forces of return. The force of return is the continuously compounded return and is given by the natural logarithm of the gross return.

Equation 5 The force of return

$$force\ of\ return_t = ln\left(\frac{price_t}{price_{t-1}}\right)$$

Forces of return are additive over time. The annual force of return for a financial year is equal to the sum of the twelve monthly forces of return that fall within that year. This additive structure is also important for the reshuffling procedure used later, as it allows annual forces of return to be reconstructed from alternative sequences of monthly observations.

Equation 6 Conversion from monthly to annual forces of return

$$annual\ force\ of\ return = \sum_{i=1}^{12} monthly\ force\ of\ return_i$$

Figure 10 plots the resulting monthly forces of return for the Vanguard Australian Fixed Interest Index Fund and to the S&P ASX Australia Aggregate Bond Index. Over their overlapping period the two series move closely together, with similar patterns in both directions. The S&P index tends to show slightly higher positive peaks and slightly less negative troughs than the Vanguard fund, which is consistent with the presence of management fees and small tracking differences in the Vanguard unit trust. Overall, the close correspondence between the two series supports the use of the Vanguard fund net asset values as a reasonable proxy for the missing historical data of the S&P ASX Australia Aggregate Bond Index in the earlier years of the sample.

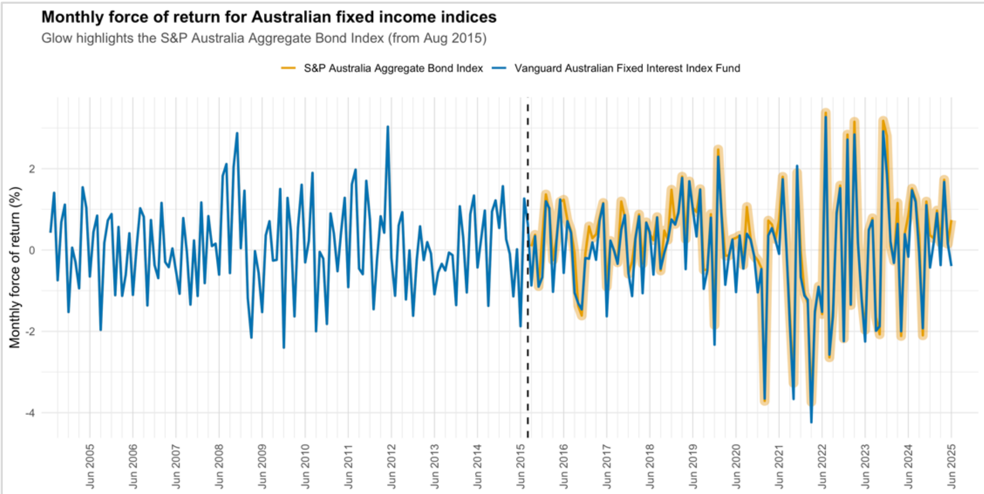

Figure 10: Comparing Australian fixed income monthly forces of return (Created by author)

Asset prices were assumed to follow a Geometric Brownian Motion (GBM) process, which is widely used in financial economics and actuarial modelling due to its analytical tractability and guarantee of strictly positive prices (Akansu & TorunTorun, 2015). Under GBM, continuously compounded returns are normally distributed, and normal distributions were therefore fitted to the annual forces of return for each asset class. While this assumption is empirically inconsistent with observed financial returns (Nkemnole & Abass, 2019), which exhibit skewness, excess kurtosis, and time-varying volatility, it is adopted here to deliberately simplify the marginal distributions. This allows the analysis to focus on the effect of alternative dependence structures on joint tail outcomes, rather than on marginal distributional complexity.

7.2.4 Transformation to uniform scale

Before fitting any copula, the observed annual forces of return must be transformed so that each marginal distribution lies on the interval zero to one. This step is required because a copula is, by definition, a multivariate cumulative distribution function whose marginal distributions are all uniform on the interval $[0, 1]$ (Yan, 2006). The observed annual forces of return for equity, property and fixed income were therefore converted into pseudo observations on the uniform scale using their empirical ranks.

For an asset class with n annual forces of return, the transformed value is calculated as follows.

Equation 7 Transformation of forces of return to uniform scale

$$uniform\ transformed\ value = \frac{rank\ of\ observation}{n+1}$$

In Equation 7, the $n + 1$ term is a continuity correction for continuous random variables with discrete observations. Applying this formula produces uniform values strictly between zero and one while preserving the dependence structure across asset classes. Each asset class was transformed separately so that the subsequent copula fitting captured only the joint dependence structure and not the marginal behaviour.

7.2.5 Fit copula parameters

Four copula families were fitted to the transformed annual forces of return. These were the Gaussian copula, the Clayton copula and two Student's t copulas with degrees of freedom equal to three and seven. These copulas were chosen to investigate the treatment of lower tail dependence. Kole et al (2006) found that the correlation-based Gaussian copula tends to underestimate tail risks and overstate diversification benefits by failing to account for tail dependence. The Gaussian copula serves as a baseline model to compare against alternatives that better capture lower tail dependence. The Clayton copula is expected to provide the strongest tail dependence (Kole et al, 2006). The Student's t copulas with df = 7 and df = 3 provide symmetric tail dependence with increasingly heavy tails as the df parameter falls, allowing a useful comparison for how strongly results change when joint extremes are more likely. Degrees of freedom 7 and 3 were chosen to represent, respectively, a realistic moderate level of tail dependence and a deliberately severe lower-tail stress, bracketing a plausible range of downside dependence without introducing an additional estimated parameter.

The transformation to the uniform scale described in step 4 served as the input to the copula estimation step. The Gaussian and Clayton copulas were estimated using the inversion of Kendall tau method. This estimation method provides closed form, rank-based parameter estimates that are well suited to small samples and that avoid the numerical instability that can arise from optimisation based procedures. The Student's t copula includes an additional parameter, the degrees of freedom, which determines the strength of joint tail dependence and cannot be estimated using Kendall tau inversion. If Kendall tau inversion were used, the Student's t copula would behave essentially like a Gaussian copula with a fixed degrees of freedom value and would lose its ability to represent heavy tailed dependence. For this reason the Student's t copulas were estimated using maximum likelihood in order to allow the degrees of freedom parameter to be fitted.

The sample of annual forces of return contains only 21 observations, and the copula parameters are therefore estimated with limited information. The choice of estimation method was made with this constraint in mind, so that parameter fitting remained stable and so that the Student's t copula retained its tail dependence structure. The estimation procedure was implemented using the copula package in R.

7.2.6 Simulate from fitted copula

Once the copula parameters were estimated, each copula was used to generate simulated joint scenarios for equity, property and fixed income. For each copula family, 10,000 simulated triplets of uniform values were produced. These simulated values represent the dependence structure implied by the fitted copula but have no marginal interpretation until they are transformed back into annual forces of return in the next step.

For some copula families, particularly the Clayton and Student's t copulas, certain simulated values were missing. In the Clayton case, this occurred because the Clayton copula is only defined for positive dependence. When the historical annual data exhibit weak or negative dependence, the parameter estimated from the sample may fall close to zero or outside the valid range, which can lead to invalid or undefined simulated values. These invalid outputs were identified and removed before any percentiles or summary statistics were calculated.

For the Student's t copula, missing values typically arose from numerical difficulties in the maximum likelihood fitting procedure. With only 21 annual observations, the likelihood surface can be flat or difficult to optimise, especially for the copula with seven degrees of freedom where the dependence structure approaches that of the Gaussian copula. In these cases the fitted parameter values may be invalid or may not correspond to a positive definite correlation matrix. Simulations based on such invalid fits generate missing values. These were also removed before any post processing. The objective of this step was to ensure that only valid simulated scenarios were carried forward to the subsequent calculations.

7.2.7 Convert simulated values into annual forces of return

The simulated values obtained from the copulas lie on the interval zero to one and must be transformed into economically meaningful annual forces of return. This was done by applying the inverse normal distribution to each simulated value. The mean and variance parameters of the normal distribution were those estimated from the historical annual forces of return for each asset class in step 3. This transformation ensures that the marginal behaviour of each simulated scenario is consistent with the observed annual data while the dependence across asset classes follows the structure implied by the fitted copula.

This step was also the main motivation for adopting the Geometric Brownian Motion assumption for asset prices. Under this assumption, the forces of return are normally distributed. With only 21 annual observations, a nonparametric approach based on the empirical distribution function would produce unstable inverse transforms, particularly in the tails where very few observations are available and where interpolation is required. A continuous normal distribution provides stable and well defined quantile values across the entire zero to one range and allows the copula simulated values to be converted into forces of return in a consistent and reliable way.

The result of this final step is a simulated joint distribution of annual forces of return for equity, property and fixed income that incorporates both the fitted marginal behaviour and the estimated dependence structure.

7.2.8 Joint surplus at t=1

For each simulated set of annual forces of return for equity, property and fixed income, the corresponding value of the insurer's assets at time one was calculated by applying the exponential of each simulated force to the relevant asset value at time zero. The notional balance sheet used for this purpose is shown in table 12 and contains four asset categories: equity, property, fixed income and a residual category labelled other assets. The other assets component represents all remaining asset holdings of the reference insurer that are not part of the three market sensitive asset classes considered in this modelling exercise. The focus of the thesis is the effect of market risk on the insurer's financial position, and the values of other assets are therefore held constant throughout.

The total liabilities were also held constant over the one year projection period. This assumption isolates the impact of asset side market movements on the insurer's financial position and is consistent with the scope of the study, which is limited to the modelling of market risk rather than liability valuation.

For a simulated scenario with annual forces of return denoted by equity force, property force and fixed income force, the simulated surplus at time one was calculated according to Equation 8.

Equation 8 Calculation of the joint surplus at t=1

$$joint\ surplus_{t=1} = equity_{t=0} \times e^{equity\ force} + property_{t=0} \times e^{property\ force} + fixed\ income_{t=0} \times e^{fixed\ income\ force} + other\ assets_{t=0} - total\ liabilities_{t=0}.$$

This formula combines the simulated performance of the three market sensitive asset classes in a way that preserves their copula-estimated dependence structure, while keeping the remaining notional balance sheet items unchanged. It produces one simulated surplus value for each simulated copula triplet. The capital requirement that can be compared to the standard method approach is the corresponding fall in the fund's capital base over one year. This can be expressed as the $fall\ in\ capital\ base = surplus_{t=0} - joint\ surplus_{t=1}$, where the surplus at t=0 was 100.

7.2.9 The 0.5 percentile of copula-generated joint surpluses

In Australia, the PCA is calibrated to ensure that a life insurer has a 99.5 per cent probability of meeting all obligations over the following year. This requirement is assessed by examining the left tail of the distribution of possible surplus outcomes and identifying the level of surplus that would remain under the most adverse 0.5 per cent of scenarios. In this thesis the 10,000 simulated surplus values generated for each copula family were therefore summarised by their 0.5 percentile. This percentile represents the surplus that would remain after a 1-in-200 year market shock, given the marginal behaviour and dependence structure captured by the fitted copula.

7.2.10 Reshuffled monthly forces of return and construction of alternative annual scenarios

The historical data was limited to only 21 annual observations, which leads to substantial uncertainty in the estimation of copula dependence parameters and restricts the range of adverse outcomes that can be used to assess tail risk. To explicitly illustrate the impact of this data sparsity on dependence calibration and resulting capital estimates, an additional modelling step was introduced in which the monthly forces of return were reshuffled to create alternative but equally plausible historical sequences of annual returns. This is possible due to the additive property of forces of return, whereby the sum of monthly forces of return is equal to the corresponding annual force of return. The reshuffling preserves the empirical distribution of monthly forces of return while generating alternative historical paths that could reasonably have occurred had a longer return history been available. The objective is to assess how the estimated 0.5 percentile of the joint surplus after one year varies as copula parameters are re-estimated under different plausible historical sequences, thereby producing a range of plausible 1-in-200 outcomes rather than a single point estimate.

For each reshuffle, the 252 historical monthly forces of return were randomly rearranged, while keeping each month's equity, property and fixed income observations together so that the cross sectional dependence within each month was retained. The reshuffled series was then segmented into consecutive blocks of twelve months. The monthly forces of return within each block were summed to produce an annual force of return. This procedure yielded a set of 21 annual forces of return for each random reshuffle, matching the number of historical financial years in the original dataset.

The full copula-based modelling pipeline described in the previous sections (steps 3 to 9 in Figure 9) was applied to each reshuffled dataset, with copula parameters re-estimated separately for each reshuffle. For every reshuffle, 10,000 simulated triplets of annual forces of return were generated from the fitted copula, transformed into simulated forces of return, and converted into 10,000 simulated joint surplus outcomes at time t=1. The 0.5 percentile of these simulated joint surplus values was recorded as the estimate of the 1-in-200 year worst outcome for that particular reshuffle.

This process was repeated for 10,000 independent reshuffles, producing 10,000 estimates of the 0.5 percentile of the simulated joint surplus for each copula type. The resulting distribution of these estimates represents a band of plausible 1-in-200 outcomes arising solely from dependence parameter uncertainty induced by limited historical data. These distributions form the basis for comparing the behaviour of the Gaussian, Student's t and Clayton copulas under a wide range of plausible historical return sequences. The boxplot in Figure 14 illustrates the median and interquartile range of these distributions for each copula type, highlighting the sensitivity of the estimated 1-in-200 surplus outcome to both the chosen dependence model and the sequencing of historical returns. Density plots of these distributions are presented in Figure 15 in Section 8.2.2 to provide further insight into the shape and tail behaviour of the estimated percentiles.

7.3 Standard method approach

To construct a benchmark capital requirement under the Australian Standard Method that is comparable to the copula joint surplus results, the asset risk charge was approximated for a notional balance sheet by separately calculating the equity, property and fixed income (real interest rate) risk charge components.

Each component of the asset risk charge was calibrated to achieve a 99.5 per cent probability of sufficiency over a one-year horizon. The capital charge for each asset risk charge component is represented as the fall in the notional insurer's surplus associated with a 1-in-200 adverse outcome over one year. This approach of calibrating each market risk component separately is consistent with Solvency II, under which each market risk sub-module is calibrated to a 99.5 per cent one-year Value-at-Risk prior to aggregation. Although the Australian Prudential Regulation Authority (APRA) does not explicitly state that individual asset risk charge components correspond to a standalone 99.5 per cent probability of sufficiency, the Australian standard method is regarded as closely aligned with Solvency II. Treating each asset class stress as a 1-in-200 adverse outcome therefore provides a conceptually consistent benchmark for comparison with the copula-based capital requirements.

To isolate market risk and ensure comparability with the copula-based joint surplus distribution, only equity, property and real interest rate risk charge components were considered. Other asset risk charge components, including default risk, currency risk, credit spread risk and expected inflation risk, were excluded.

The 1-in-200 adverse outcome for each asset class was determined using fitted normal distributions of annual forces of return. Distribution parameters were estimated using 21 historical annual forces of return, and the 0.5 per cent quantile of each fitted distribution was calculated to represent the 1-in-200 worst-case annual force.

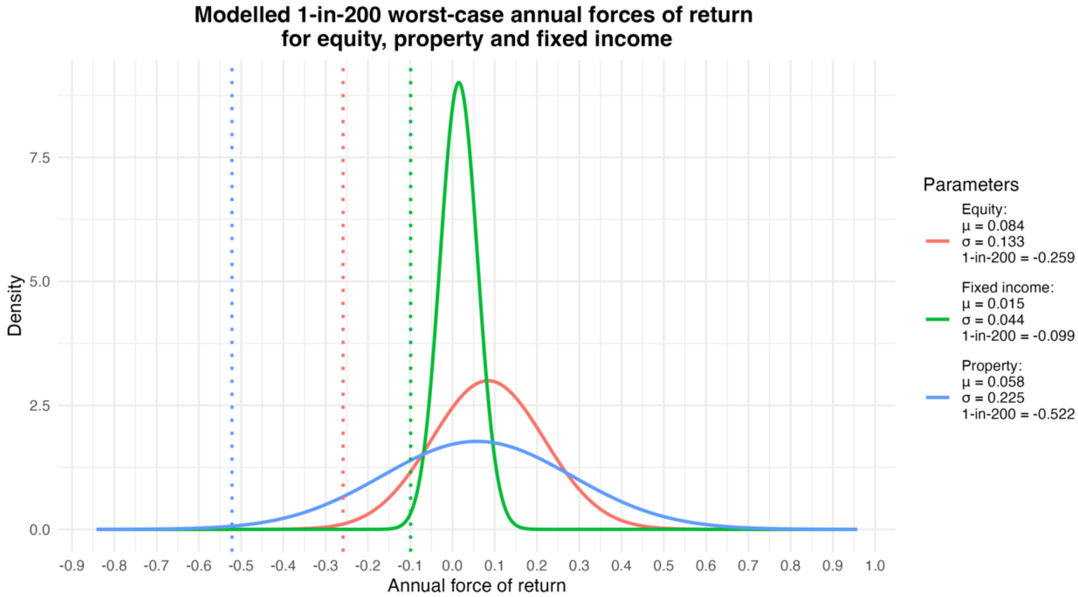

Figure 11 Comparing the fitted normal distributions and 0.5% quantile of the equity, property and fixed income annual forces of return (Created by author)

Marginal surplus at time t = 1 was then calculated separately for each asset class by applying the relevant 0.5 per cent quantile force while holding all other balance sheet components constant, consistent with the standard method's marginal risk charge framework. For example, the stressed surplus under an equity shock is given by $equity\ surplus_{t=1} = equity_{t=0} \times e^{0.5\ quantile\ equity\ force} + property_{t=0} + fixed\ income_{t=0} + other\ assets_{t=0} - total\ liabilities_{t=0}$. The equity capital charge is defined as the reduction in total surplus resulting from the stress, where initial surplus is normalised to 100. The resulting capital requirements are reported in Section 8.2.

8. Results

8.1 Standard method results

Using the standard method approach outlined in section 7.3, the 1-in-200 worst force of return for each asset class in the notional balance sheet, and the resulting capital charges for each component of the asset risk charge are summarised in table 13.

Table 13 Standard method calculation of individual risk charges

Asset class	1-in-200 worst force of return	Marginal surplus (t=1)	Risk charge (A_i)
Equity	-25.87459 %	97.405	2.594999
Property	-52.15771 %	70.82732	29.17268
Fixed Income	-9.90054 %	56.41899	43.58101

The results from cross-multiplying the risk charge components for the stress tests including the correlation factors and the "sign" functions are shown in table 14.

Table 14 Application of the Australian asset risk charge aggregation formula

	Equity	Property	Fixed Income
Equity	$2.595 \times 2.595 \times 1.00 \times 1 \times 1 = 6.7340$	30.2812	22.6185
Property	$2.595 \times 29.173 \times 0.4 \times 1 \times 1 = 30.2812$	851.0453	254.2750
Fixed Income	$2.595 \times 43.581 \times 0.2 \times 1 \times 1 = 22.6185$	254.2750	1899.3044

The aggregation is found by adding the numbers in the table (3371.4332), taking the square root (58.0640), and then adding the default charge (assumed 0 in this case). Therefore, following the standard method, the capital requirement for market risk for the notional insurer is 58.064. To compare with the copula-generated joint surplus values, a capital requirement of 58.064 corresponds to a joint surplus at t=1 of 41.936.

8.2 Copula modelling results

8.2.1 Results from using historical annual forces

First, each copula was fitted to the 21 historical annual forces of return for each asset class. Although each copula was calibrated to the same input data, the resulting 1-in-200 joint surplus and implied fall in capital base differed materially across copula families, as outlined in table 15, highlighting the sensitivity of capital outcomes to the assumed dependence structure. This demonstrates that the choice of copula is a key modelling decision with direct regulatory capital implications.

Table 15 Copulas fitted using the historical annual forces of return. Parameters and 0.5% VaR of joint surplus at t=1

Copula type	Parameters	0.5% VaR of the joint surplus (t=1)	1-in-200 worst fall in capital base
Gaussian	$\rho_{equity,property} = 0.64659960$ $\rho_{equity,fixed\ income} = -0.44731315$ $\rho_{property,fixed\ income} = 0.01495941$	49.80888	50.19112
Student's t (df=7)	$df = 7$ $\rho_{equity,property} = 0.69869995$ $\rho_{equity,fixed\ income} = -0.38363226$ $\rho_{property,fixed\ income} = -0.04004299$	46.38885	53.61115
Student's t (df=3)	$df = 3$ $\rho_{equity,property} = 0.65145337$ $\rho_{equity,fixed\ income} = -0.36196280$ $\rho_{property,fixed\ income} = -0.01750262$	43.35018	56.64982
Clayton	$\theta = 0.3946794$	37.22735	62.77265

The Gaussian copula produced the highest 0.5% joint surplus and therefore the lowest fall in the notional insurer's capital base. Its dependence parameters are correlation coefficients of latent normal variables and imply symmetric dependence with zero tail dependence. As a result, extreme joint downside outcomes are relatively muted, leading to lower capital requirements.

The Student's t copulas introduced tail dependence while retaining correlation-based dependence parameters that are directly comparable to the Gaussian case. Despite broadly similar ρ values across models, reducing the degrees of freedom materially lowers the 0.5% joint surplus and increases the fall in capital base. This reflects stronger joint downside risk driven by heavier tails rather than changes in overall dependence strength. The comparison between df = 7 and df = 3 illustrates that tail behaviour, not correlation magnitude, is the dominant driver of capital outcomes in the extreme quantile.

The dependence parameter ρ in the Gaussian and Student's t copulas represents the correlation between latent elliptical variables underlying the copula construction. While the parameter is directly comparable across copula families and lies on the same scale, identical values of ρ do not imply identical joint tail behaviour. In particular, Student's t copulas exhibit non-zero tail dependence, with the strength of joint extreme outcomes increasing as the degrees of freedom decrease. Consequently, for a given value of ρ, a t-copula with lower degrees of freedom implies materially stronger joint downside risk than either a t-copula with higher degrees of freedom or a Gaussian copula.

The Clayton copula produced the most conservative result, with the lowest 0.5% joint surplus and largest fall in capital base. Its single parameter θ is not a correlation and is not directly comparable to the ρ parameters of elliptical copulas. Instead, θ governs asymmetric lower-tail dependence, explicitly increasing the probability of joint adverse outcomes. This behaviour aligns with economic intuition for asset returns, where co-movement tends to strengthen during market stress, and leads to materially higher capital requirements.

Overall, these results demonstrate that models with stronger lower-tail dependence generate higher 1-in-200 capital requirements, even when calibrated to the same historical data. From a regulatory perspective, this underscores the importance of selecting a copula family that appropriately reflects downside dependence, as standard correlation-based models may understate extreme joint risk for life insurers.

Out of the four copula types, only the Clayton copula's fall in capital base and resulting capital requirement, 62.77 (2dp), was higher than the capital requirement determined using the standard method in section 8.1, 58.06 (2dp).

8.2.2 Distribution of results from reshuffled annual forces

Repeating the copula fitting pipeline using reshuffled forces of return resulted in a distribution of the 0.5% Value at Risk of the joint surplus at t=1. As illustrated in Figure 12, the interquartile ranges a cross all copula families are large, indicating substantial variability in the one-year 0.5% Value at Risk of the joint surplus outcome purely due to alternative historical sequencing. This provides strong justification for the reshuffling approach, as relying on a single realised history would give a misleading impression of precision in tail risk estimates.

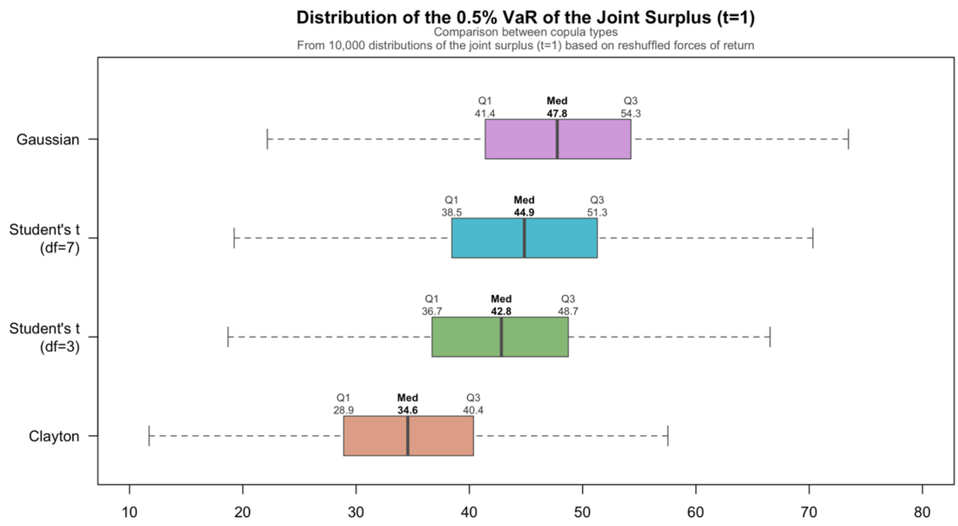

Figure 12: Comparison between copula types of the distribution of the 0.5% VaR of the joint surplus (t=1). (Created by author)

As outlined in table 16 and illustrated in Figure 15, the Gaussian copula produced the highest median and upper-quartile joint surplus values, and therefore the lowest implied capital requirement. However, it still exhibited a wide range between the minimum and maximum outcomes, showing that even under symmetric dependence, adverse sequencing can materially worsen capital outcomes.

Table 16 Summary statistics of the reshuffled 0.5% VaR of the joint surplus (t=1) across copula types

Copula type	Minimum	1st quartile	Median	Mean	3rd quartile	Maximum	NA's
Gaussian	13.58	41.41	47.77	47.77	54.27	81.62	*0*

Student's t (df=7)	12.29	38.45	44.85	44.96	51.29	77.34	*2981*
Student's t (df=3)	10.03	36.71	42.83	42.74	48.74	74.37	*2372*
Clayton	6.03	28.90	34.57	34.65	40.36	66.16	*6*

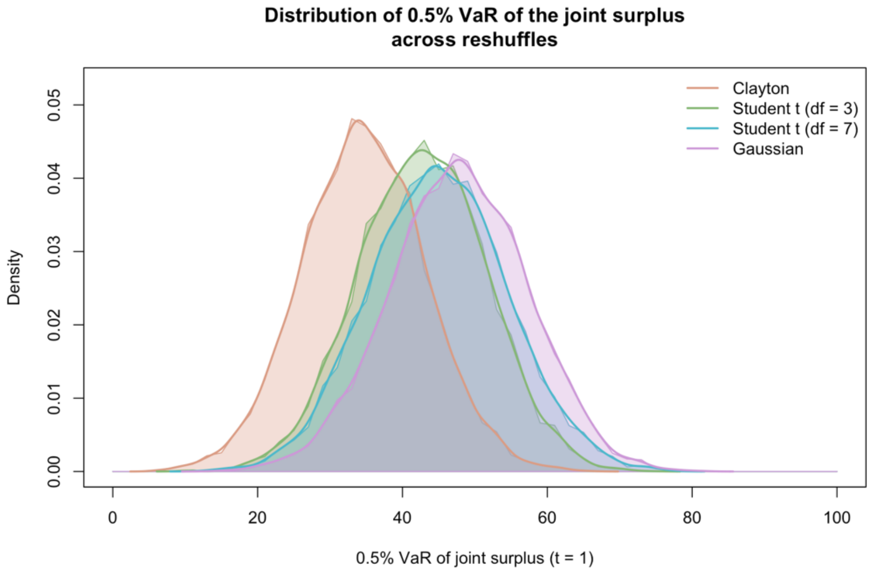

Figure 13 Density plots comparing the distributions of the joint surplus (t=1) across copula families (Created by author)

Both the df = 7 and df = 3 Student's t copulas produced systematically lower medians and quartiles of the 0.5% Value at Risk of the joint surplus than the Gaussian copula. In other words, both Student's t copulas implied higher capital requirements. The smaller joint surplus value is more pronounced for df = 3, consistent with stronger tail dependence. Importantly, the lower quartiles of the joint surplus 0.5% Value at Risk shift materially downward while upper quartiles remain closer to the Gaussian case, indicating that tail behaviour rather than central dependence is driving the difference. The Student's t copulas capture a higher probability of more extreme lower tail outcomes of the joint surplus.

The Clayton copula produced the lowest median and first-quartile joint surplus outcomes, with an especially severe minimum of 6.03. This reflects the explicit modelling of lower-tail dependence, which increases the likelihood of joint adverse outcomes under stressed sequences. The wide range between minimum and maximum 0.5% VaR of the joint surplus values further suggests that downside outcomes are highly sensitive to historical sequencing when asymmetric dependence is assumed.

9. Conclusion

In answer of the first research question, regulatory capital requirements for market risk are determined using broadly similar principles across jurisdictions, but with important structural differences. Australia and the European Union both employ stress-based market risk charges calibrated to a 99.5 per cent one-year probability of sufficiency, with individual asset or market risk components aggregated using prescribed correlation matrices to allow for diversification benefits. In contrast, the United States and Canada adopt factor-based or mixed stress- and scenario-based approaches and do not permit diversification within market risk to the same extent. This comparison highlights that Australia's Standard Method is closely aligned with Solvency II in its reliance on correlation-based aggregation, while differing materially from North American frameworks in both structure and underlying assumptions.

Secondly, copula-based dependence modelling provides a flexible alternative framework for analysing the interaction between market risk drivers, particularly in the tails of the joint distribution. By separating marginal distributions from the dependence structure, copulas allow non-linear and tail-dependent relationships to be modelled explicitly, which cannot be achieved using linear correlation alone. The Gaussian, Student's t and Clayton copulas considered in the modelling and results section span a range of dependence assumptions, from no tail dependence to strong lower-tail dependence, which enabled a targeted exploration of joint downside risk. A copula-based method can be adopted as an exploratory or complementary tool that can enhance understanding of dependence-driven tail outcomes rather than as a direct replacement for current regulatory standard methods.

Finally, using a copula-based joint distribution did materially alter the estimated market risk capital requirements relative to the Australian Standard Method approach. However, the effect depended on the assumed dependence structure. Under the Standard Method, the 0.5 per cent Value at Risk of the joint surplus was estimated to be 41.936, corresponding to a capital requirement of 58.064 over one year. When copulas were calibrated directly to the 21 historical annual forces of return, the implied capital requirements were 50.191 for the Gaussian copula, 53.611 for the Student's t copula with 7 degrees of freedom, 56.650 for the Student's t copula with 3 degrees of freedom, and 62.773 for the Clayton copula. Interestingly, only the Clayton copula produced a capital requirement exceeding that implied by the Standard Method, reflecting the impact of explicit lower-tail dependence rather than symmetric heavy tails alone.

When historical sequencing uncertainty was incorporated through the reshuffling of monthly returns, the resulting distribution of 0.5 per cent joint surplus estimates widened substantially. Across 10,000 reshuffles per copula, the Clayton copula produced the most adverse outcomes, with joint surplus values ranging from a minimum of 6.03 to a maximum of 66.16, indicating pronounced sensitivity to sequencing under asymmetric dependence. These results demonstrate that dependence assumptions and data limitations can have a first-order effect on estimated 1-in-200 outcomes, and that correlation-based aggregation may mask substantial downside risk when lower-tail dependence is present.

A key limitation of this analysis is the reliance on a short historical sample of only 21 annual observations, which leads to substantial uncertainty in copula parameter estimation and occasional numerical instability in maximum likelihood fitting, particularly for heavy-tailed and asymmetric copulas. Additionally, the reshuffling step in the copula modelling approach illustrated sensitivity to historical sequencing rather than providing formal statistical confidence intervals, and results should therefore be interpreted as indicative of plausible ranges rather than precise capital estimates. Finally, simplifying assumptions regarding marginal return distributions and balance sheet dynamics were adopted to isolate dependence effects, which may understate or overstate absolute capital levels but do not detract from the comparative insights on dependence structure.

10. References

AIHW. (2024). *Demographic profile of older Australians*. https://www.aihw.gov.au/reports/older-people/older-australians/contents/demographic-profile

Akansu, A. N., & TorunTorun, M. U. (2015). *A Primer for Financial Engineering: High Frequency Trading and Signal Processing Models for the Microstructure of Financial Markets*. https://www.sciencedirect.com/topics/engineering/geometric-brownian-motion

APRA. (2010). *Discussion Paper: Review of capital standards for general insurers and life insurers*. https://www.apra.gov.au/sites/default/files/GLI_DP_RCSGILI_032010_v7%5B1%5D.pdf

APRA. (2010). *Technical Paper: Review of capital standards for general insurers and life insurers - Asset risk capital charge*. https://www.apra.gov.au/sites/default/files/GLI_TP_LGICR_062010_ex%5B1%5D.pdf

APRA. (2011). *Response to Submissions: Review of capital standards for general insurers and life insurers*. https://www.apra.gov.au/sites/default/files/GLI_RS_RCSGILI_032011_ex_r%5B1%5D_0.pdf

APRA. (2011). *Response to Submissions: Review of capital standards for general insurers and life insurers*. https://www.apra.gov.au/sites/default/files/RP_RCSGILI_122011_v17_0.pdf

APRA. (2013). *Prudential Standard LPS 110 Capital Adequacy*. https://www.apra.gov.au/sites/default/files/LPS-110-Capital-Adequacy-January-2013_0.pdf

APRA. (2013). *Information Paper: Asset Risk Charge*. https://www.apra.gov.au/sites/default/files/information-paper-asset-risk-charge-march-2013_0.pdf

APRA. (2015). *Prudential Standard LPS 001 Definitions*. https://www.apra.gov.au/sites/default/files/prudential_standard_gps_001_definitions_2.pdf

APRA. (2023). *Reporting Standard LRS 110.0 Prescribed Capital Amount*. https://www.legislation.gov.au/F2023L00627/latest/text

APRA. (2023). *Prudential Standard LPS 114 Capital Adequacy: Asset Risk Charge*. https://www.legislation.gov.au/F2023L00676/latest/text

APRA. (2025). *Significant financial institutions register*. Australian Prudential Regulation Authority. https://www.apra.gov.au/significant-financial-institutions-register

APRA. (2025). *Quarterly life insurance performance statistics*. https://www.apra.gov.au/quarterly-life-insurance-performance-statistics

APRA. (2025). *APRA confirms consultation on changes to capital requirements for annuity products*. https://www.apra.gov.au/news-and-publications/apra-confirms-consultation-on-changes-to-capital-requirements-for-annuity

APRA. (Undated). *CPPP Investment indices*. Australian Prudential Regulation Authority. 2025, from https://www.apra.gov.au/cppp-investment-indices

Austin, R., Ashcroft, M., Dick, C., Morgan, S., Macpherson, A., McBride, A., Marshall, I., & Taylor, R. (2009). *Practical Implementation Challenges of Internal Models under Solvency II*. Faculty of Actuaries Solvency & Capital Management Research Group. https://sias.org.uk/media/1164/practical-implementation-challenges-of-internal-models-under-solvency-ii.pdf

Campbell, I. (2018). *Why insurers fail: Lessons learned from the failure of HIH Insurance Limited*. Property and Casualty Insurance Compensation Corporation. https://www.pacicc.ca/wp-content/uploads/2018/04/2018-Why-Insurers-fail_E_Full.pdf

Challenger. (2025). *Challenger Limited FY25 Analyst Pack*. https://edge.sitecorecloud.io/challengerg66c7-challengerprod-prod-4c11/media/Project/Challenger/Challenger/Documents/Packs/2025-Analyst-Pack.pdf

Embrechts, P., McNeil, A., & Straumann, D. (1999). Correlation and Dependence in Risk Management: Properties and Pitfalls. https://people.math.ethz.ch/~embrecht/ftp/pitfalls.pdf

European Parliament, C. o. t. E. U. (2009). *Directive 2009/138/EC of the European Parliament and of the Council of 25 November 2009 on the taking-up and pursuit of the business of Insurance and Reinsurance (Solvency II) (recast) (Text with EEA relevance)*. European Parliament, Council of the European Union. https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX%3A32009L0138

Fan, Y., & Patton, A. J. (2014). Copulas in Econometrics. *Annual Review of Economics*, *6*, 179-200. https://doi.org/https://doi.org/10.1146/annurev-economics-080213-041221

Field, A. (2015). *Solvency II's coming challenges*. Reinsurance Group of America. https://www.rgare.com/knowledge-center/article/solvency-ii-s-coming-challenges

Genest, C., & Favre, A.-C. (2007). Everything You Always Wanted to Know About Copula Modelling but were Afraid to Ask. *Journal of Hydrologic Engineering*, *12*(4), 347-368. https://www.uni-muenster.de/Physik.TP/~lemm/seminarSS08/JHE-2007.pdf

Jan, Y. (2006). *Springer Handbook of Engineering Statistics: Multivariate Modeling with Copulas and Engineering Applications*. Springer, London. https://doi.org/https://doi.org/10.1007/978-1-84628-288-1_51

Kendall, M. (1938). A New Measure of Rank Correlation. *Biometrika*, *30*, 81-89. https://doi.org/https://doi.org/10.1093/biomet/30.1-2.81

Kirk, D. (2025). *What's in a tail? Risk doesn't stop at VaR – what CTE captures that VaR ignores*. https://za.milliman.com/en-GB/insight/whats-in-a-tail-var-risk-cte

Kole, E., Koedijk, K., & Verbeek, M. (2005). Testing copulas to model financial dependence. https://api.semanticscholar.org/CorpusID:19012940

Kole, E., Koedijkb, K., & Verbeekb, M. (2006). Selecting Copulas for Risk Management. https://repub.eur.nl/pub/12668/SelectingCopulas_2005.pdf

Leiser, B., Bender, J., & Kaul, B. (2023). Regulatory Capital Adequacy for Life Insurance Companies: A Comparison of Four Jurisdictions. *Society of Actuaries Research Institute*. https://www.soa.org/4a194f/globalassets/assets/files/resources/research-report/2023/erm-191-reg-capital-with-final-visuals.pdf

McCullough, K. A. (2014). Basis Risk , Procyclicality , and Systemic Risk in the Solvency II Equity Risk Module. https://api.semanticscholar.org/CorpusID:164215910

McDonald, R., & Paulson, A. (2015). *What Went Wrong at AIG? Unpacking the insurance giant's collapse during the 2008 financial crisis.* Kellogg School of Management, Northwestern University. https://insight.kellogg.northwestern.edu/article/what-went-wrong-at-aig

Navratil, P. (2024). *Correlation in Capital Frameworks: Comparison of Correlation in Other Regulatory Capital Frameworks, Life Investment and Capital Adequacy Committee*. American Academy of Actuaries. https://content.naic.org/sites/default/files/call_materials/Life-Presentation-CorrelationLRBC.pdf

Necula, C. (2010). Modeling the Dependency Structure of Stock Index Returns using a Copula Function Approach. *Romanian Journal of Economic Forecasting*, 93-106. https://api.semanticscholar.org/CorpusID:19984251

Nelsen, R. B. (2006). *An Introduction to Copulas* (Second ed.).

Nkemnole, B., & Abass, O. (2019). Estimation of geometric Brownian motion model with a t-distribution–based particle filter. *Journal of Economic and Financial Sciences*, *12*(1). https://doi.org/10.4102/jef.v12i1.159

OSFI. (2025). *Life Insurance Capital Adequacy Test (2025) – Chapter 11 Aggregation and Diversification of Risks*. https://www.osfi-bsif.gc.ca/en/guidance/guidance-library/life-insurance-capital-adequacy-test-2025-chapter-11-aggregation-diversification-risks

Schmidt, T. (2007). Coping with Copulas. *Copulas - From Theory to Application in Finance*. https://www.researchgate.net/publication/228876267_Coping_with_copulas

Sherris, M., & Hoek, J. v. d. (2008). *Fitting and Estimating Risk Dependence using Copulas for Multivariate Data*. Risk and Capital Management Research Conference, Sydney, Australia. https://actuaries.asn.au/Library/Michael%20Sherris%20-%20RC08SherrisvanderHoeklatest.pdf

Spearman, C. (1904). The Proof and Measurement of Association between Two Things. *The American Journal of Psychology*, *15*(1), 72-101. https://doi.org/https://doi.org/10.2307/1412159

Venter, G., Barnett, J., Kreps, R., & Major, J. (2007). Multivariate Copulas for Financial Modeling. *Variance*, *0001,0001,Spring*, 0103-0119. https://www.casact.org/sites/default/files/2021-07/Multivariate-Copulas-Venter-Barnett-Kreps-Major.pdf

Venter, G. G. (2002). Tails of Copulas. *Casualty Actuarial Society*, *LXXXIX*, 68-113. https://www.casact.org/sites/default/files/database/proceed_proceed02_02068.pdf

Venter, G. G. (2003). Quantifying Correlated Reinsurance Exposures with Copulas. *Casualty Actuarial Society E-Forum*, *Spring*, 215-229. https://www.casact.org/sites/default/files/database/forum_03spforum_03spf215.pdf

Wiehenkamp, C. (2010). Life Insurance and the Agency Conflict: An Analysis of Prudential Regulation to Guard Policyholders from Excessive Risk Taking. *Household Finance eJournal*. https://doi.org/https://doi.org/10.2139/ssrn.1605086

Note that AI tools were used for grammatical edits and for parts of the literature review.

Appendix

Appendix A: general coefficient of copula upper and lower tail dependence

Summary table: general coefficient of upper and lower tail dependence		
Copula dimension	**Lower tail (λ_L)**	**Upper tail (λ_U)**
2D	$\lambda_L = \lim_{u \to 0^+} \frac{C(u,u)}{u}$	$\lambda_U = \lim_{u \to 1^-} \frac{1 - 2u + C(u,u)}{1-u}$
3D	$\lambda_L^{(3)} = \lim_{u \to 0^+} \frac{C(u,u,u)}{u}$	$\lambda_U^{(3)} = \lim_{u \to 1^-} \frac{1 - 3u + \Sigma_{i>j} C_{ij}(u,u) - C(u,u,u)}{1-u}$

Appendix B: copula distribution functions and generator functions

Summary table: copulas distribution functions and generator functions		
Copula Family (dimension)	**Distribution function**	**Parameter**
Gaussian (d>2)	$C[u_1, \dots, u_d;\, \sum] = \phi_{\Sigma}\, [\phi^{-1}(u_1), \dots, \phi^{-1}(u_d)]$	$\Sigma \in [-1, 1]$
Student's t (d>2)	$C[u_1, \dots, u_d;\, \sum] = \mathrm{T}_{\Sigma\,,v}\, [{F_{t_v}}^{-1}(u_1), \dots, {F_{t_v}}^{-1}(u_d)]$	$\Sigma \in [-1, 1]$ $\nu > 0$
Clayton (d>2)	$C[u_1, \dots, u_d;\, \theta] = ({u_1}^{-\theta} + \dots + {u_d}^{-\theta} - d + 1)^{-\frac{1}{\theta}}$ $\varphi(t) = \frac{(t^{-\theta} - 1)}{\theta}$	$\theta \in (0, \infty)$
Gumbel (d>2)	$C[u_1, \dots, u_d] = e^{-[(-\ln u_1)^{\theta} + \dots + (-\ln u_d)^{\theta}]^{\frac{1}{\theta}}}$ $\varphi(t) = (-\ln t\,)^{\theta}$	$\theta \in [1, \infty]$
Frank (d>2)	$C[u_1, \dots, u_d] = -\frac{1}{\theta} \ln(1 + \frac{(e^{-\theta u_1} - 1) \dots (e^{-\theta u_d} - 1)}{(e^{-\theta} - 1)^{d-1}}$ $\varphi(t) = -\ln(\frac{e^{-\theta t} - 1}{e^{-\theta} - 1})$	$\theta \in \mathbb{R}, \theta \neq 0$

Appendix C: closed form tail dependence coefficients for common copulas

Summary table: closed form tail dependence coefficients for common copulas			
Copula	**Lower tail**	**Upper tail**	**Notes**
Gaussian (2D)	$\lambda_L = 0$	$\lambda_U = 0$	Symmetric, no tail dependence
Gaussian (3D)	$\lambda_L^{(3)} = 0$	$\lambda_U^{(3)} = 0$	Symmetric, no tail dependence
Student's t (2D)	$\lambda_L = 2t_{\nu+1}\left(-\sqrt{\frac{(\nu+1)(1-\theta)}{1+\theta}}\right)$	$\lambda_U = \lambda_L$ (symmetric)	Symmetric, df allows heavier tails
Student's t (3D)	No simple closed form	No simple closed form	Symmetric, df allows heavier tails
Clayton (2D)	$\lambda_L = 2^{-\frac{1}{\theta}}$	$\lambda_U = 0$	Asymmetric, lower tail dependence
Clayton (3D)	$\lambda_L^{(3)} = 3^{-\frac{1}{\theta}}$	$\lambda_U^{(3)} = 0$	Asymmetric, lower tail dependence
Gumbel (2D)	$\lambda_L = 0$	$\lambda_U = 2 - 2^{\frac{1}{\theta}}$	Asymmetric, upper tail dependence
Gumbel (3D)	$\lambda_L^{(3)} = 0$	$\lambda_U^{(3)} = 3 - 3 \times 2^{\frac{1}{\theta}} + 3^{\frac{1}{\theta}}$	Asymmetric, upper tail dependence
Frank (2D)	$\lambda_L = 0$	$\lambda_U = 0$	Symmetric, higher dependence in centre
Frank (3D)	$\lambda_L^{(3)} = 0$	$\lambda_U^{(3)} = 0$	Symmetric, higher dependence in centre