Optimal Open Monetary Policy: Exchange Rate Coordination

Raymond He

ECON40018/19 Economics Research Essay

This essay is submitted in partial fulfilment of the requirements of the

Bachelor of Commerce (Honours)

October 2019

Department of Economics
The University of Melbourne

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Signed: Raymond He
Date: 20th October 2019

Acknowledgements

First and foremost, I am indebted to my supervisor Professor Chris Edmond for his invaluable guidance, kind encouragement, and unwavering support throughout the entire year. Working with and learning from you has been a privilege - I could not have asked for a better supervisor.

I have also benefited greatly from discussions with Dr James Brugler, for patiently introducing me to economic research, Dr Jiao Wang, for kindly giving her time and expertise on *Dynare* and the NOEM literature throughout the year, and Associate Professor Mei Dong, for her helpful comments and suggestions.

To my friends in the HEP '19 cohort but especially Tony Chen, Zoe Deed and Zan Fairweather - thank you for all the laughs in 137 Barry Street, proofreading early drafts and making this challenging year one that I will nostalgically remember. Special thanks also goes to Daniel Tiong and Katie Zhang - your insights and assistance from the very beginning has been crucial along every step and more important than you realise.

I am also extremely grateful for the countless hours my girlfriend Tina Cheng spent meticulously proofreading my drafts and waiting for me, and my family for their endless love and support that got me through the year. I owe them my deepest thanks for their boundless patience and understanding.

For financial support throughout this year, I would also like to acknowledge the generosity of the Australian Prudential and Regulation Authority and the Reserve Bank of Australia for awarding me the Brian Gray Scholarship.

Abstract

This paper examines whether there are gains to exchange rate coordination in a small open economy New Keynesian model with two interacting, small open economies. Both small countries (for example, Australia and New Zealand) use their nominal exchange rate against a common currency (for example, the United States Dollar) as their primary monetary policy instrument.

I make a twofold contribution to the New Open Economy Macroeconomics (NOEM) literature. Firstly, I extend the standard single small open economy - large open economy New Keynesian model by incorporating a second small open economy with which the first small open economy interacts. Secondly, I examine monetary policy cooperation with simple policy rules that central banks can implement, rather than the traditional NOEM definition of cooperation as a joint welfare-maximisation problem, which has practical shortcomings.

I find that welfare gains to exchange rate coordination are present and increase with openness to the other small open economy. However, these welfare gains peak at only 0.0034% of steady-state consumption and only emerge for countries that are highly trade-dependent and interconnected with the other cooperating country. The gains from exchange rate coordination are thus too small to advocate practical implementation between countries.

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1 Introduction

Are there welfare gains from policy coordination between monetary authorities that use the nominal exchange rate as their primary policy instrument? To answer my question, I create a model that is different from those in the NOEM literature, which has only featured either one focal small open economy among a continuum of infinitely small economies such as in Galí and Monacelli (2005), or two large open economies such as in Benigno and Benigno (2003). My model incorporates aspects from both modelling approaches by featuring two interacting small open economies that take shocks from the rest of the world (modelled as an exogenous third, large open economy) as given. The two small countries are perfectly symmetrical in every way except for the *persistence* of their total factor productivity processes. This difference allows their bilateral terms of trade to vary, and leads to different welfare outcomes between the two countries.[1]

Both small economies use their nominal exchange rate with the large economy as their monetary policy instrument in a managed float exchange rate regime. Under the non-cooperative policy, the exchange rate only responds to domestic conditions. Under the cooperative policy, both countries' exchange rates react to each others' economic conditions in addition to their own.

Single small open economy models that feature an exogenous large economy cannot facilitate coordination as the large economy does not interact with the focal economy. Two open economy models are unsuitable for examining exchange rate coordination because the two coordinating economies require a third economy to provide a common currency for their bilateral exchange rate.

I evaluate the two regimes' performance with the quadratic welfare loss function from Galí and Monacelli (2005) and find that there are indeed welfare gains to exchange rate coordination for both countries, although they are at most, 0.0034% of steady-state consumption.

[1] As explained in Section 3.2.2, countries' terms of trade would be a constant if all countries were perfectly symmetrical.

Furthermore, the country with the less persistent total factor productivity process experiences greater gains to coordination. To ensure that these gains are robust to parameter values, I firstly vary the countries' overall trade-dependence and their openness to each other.

I find that not only do welfare losses decrease as the two countries become more open and interconnected, but gains to exchange rate coordination increase too. Interestingly, welfare *losses* to coordination emerge if the two economies do not trade enough with each other. These results suggest that for a cooperative exchange rate regime to be beneficial, the two countries need to not only be highly open but also trade substantially with each other.

I also compute welfare losses under different calibrations of the exchange rate rule and find that while welfare losses decrease as the exchange rate reacts more strongly to domestic inflation and the output gap, the cooperative gains trend downwards.

Therefore, my model suggests that the gains from exchange rate coordination are too small to justify practical implementation, even for highly trade-dependent countries.

This paper brings together two research areas: literature on the welfare benefits of monetary policy coordination, and studies of the role of the exchange rate as either a target or a monetary policy instrument.

Firstly, it is well established that interest rates co-move internationally, despite countries' business cycles often being at different stages. For example, after the United States (U.S.) eased monetary policy during the Global Financial Crisis, short and long-term interest rates dropped too even in countries not directly affected by it. Moreover, as countries grow increasingly interconnected, these monetary policy spillovers are likely to worsen, which is particularly concerning for small and emerging market economies.[2] As their interest rates move in ways increasingly unrelated to their domestic conditions, their monetary authority's control over domestic conditions and ability to achieve its objectives weakens.

Monetary policy coordination has been suggested to combat and internalise international

[2] Kearns, Schrimpf and Xia (2018) finds statistically significant evidence of interest rate spillovers from 7 advanced economy central banks to 47 advanced and emerging market economies. Edwards (2015) finds significant interest rate spillovers from the U.S.

monetary policy spillovers from large economies such as the U.S. ever since Hamada (1976). Whether there are welfare gains to cooperation and, if so, how large is subject to much discussion under different models.

Secondly, the exchange rate is an important element of the transmission of monetary policy in small open economies. The literature has studied fixed and floating exchange rate regimes extensively. However, intermediate regimes, such as the managed float system, have rarely been analysed but should be more closely examined for two reasons. Firstly, the purported success of the Monetary Authority of Singapore (MAS) has shown that managed exchange rate regimes can indeed stabilise economic conditions.[3] Secondly, the proportion of International Monetary Fund (IMF) members that operate an intermediate exchange rate regime has grown from 39.5% in 2012 to 46.4% in 2018, meaning that more countries use this system than the floating exchange rate system (34.4%) IMF (2019).

The paper is organised as follows. Section 2 gives an overview of the NOEM literature. Section 3 presents the micro-founded non-policy block of the model, Section 4 closes the model by specifying both countries' monetary policy, and Section 5 calibrates the model. Section 6 presents key results and varies key parameters for sensitivity analyses. Limitations and further work are discussed in Section 7, and Section 8 concludes.

2 Literature Review

I firstly give a brief overview of the NOEM literature, followed by outlining the cooperative monetary policy literature. Finally, I provide a summary of the literature's discussion of the exchange rate's role as either a target or tool of monetary policy.

After Svensson and Wijnbergen (1989) and Obstfeld and Rogoff (1995), optimal open monetary policy is now studied using micro-founded, sticky-price models. These NOEM models allow for an accurate evaluation of different policy regimes as welfare under them

[3]See Khor, Lee, Robinson and Supaat (2007) who provides an empirical overview of Singapore's economy under a managed float exchange rate regime.

could be correctly computed, whereas the traditional Mundell (1961) model used ad hoc criteria, which may lead to spurious results.

A seminal contribution to the NOEM literature featuring Calvo (1983) pricing is Galí and Monacelli (2005), which provides the tractable framework from which most of the NOEM literature is built.[4] Domestic inflation stabilisation is optimal in their model with assumptions on parameters such as unit elasticities and log utility. However, this result does not survive if realistic departures from the model are considered, such as perfect international risk sharing and unit elasticity of substitution between domestic and foreign goods.

2.1 Gains to Cooperation

The literature has also examined optimal monetary policy design in two-country models, which create the possibility of strategic interactions and policy coordination.[5] Following Woodford (1999), policy cooperation in the NOEM literature consists of both countries delegating monetary policy to a 'supranational institution' which maximises a weighted average of the welfare of consumers in each country.

Although this approach results in the globally optimal welfare outcome, a critical drawback is that finding an analytical expression for the policy that achieved the optimal outcome is very rare, not to mention a policy that central banks can realistically implement. Even if a functional form for the policy that delivered the globally optimal outcome can be found, extreme parameter assumptions are usually required.[6]

While my ad hoc definition of cooperation does not yield the globally optimal welfare outcome, it is actionable by central banks. Thus, I also explore whether gains to cooperation are present under practical policies.

The NOEM literature examines whether gains from policy coordination are present under

[4] See Lane (2001) for a survey of earlier contributions to the NOEM literature, which often featured one period in advance price setting.

[5] Fujiwara and Wang (2017) provides a detailed taxonomy of the cooperative NOEM literature.

[6] For instance, Benigno and Benigno (2003) assumes equal international and intertemporal demand elasticities to derive exact relationship between output, consumption and relative prices.

a variety of settings. Fujiwara and Wang (2017) and Engel (2011) examine monetary policy coordination under the more empirically appealing local-currency-pricing (LCP) setting in contrast to the standard producer-currency pricing (PCP) assumption, such as in Benigno and Benigno (2003).[7] In addition to comparing non-cooperative and cooperative policy, Pappa (2004) also examines monetary unions.

While these papers all find welfare gains to cooperation, they have always been modest at best. Frictions other than nominal rigidities are needed to recommend cooperation as an important policy framework. An exception is Liu and Pappa (2008), which finds significant gains from cooperation with asymmetric trading structures.

2.2 Role of the Exchange Rate

The NOEM literature typically closes models with a simple Taylor nominal interest rate rule. The literature thus traditionally regards the exchange rate as a target. For example, De Paoli (2009) shows that under a general specification for preferences and in the presence of inefficient steady-state output, optimal policy involves targeting the real exchange rate.

However, following Parrado (2004), there is a growing literature that is now investigating the exchange rate as a monetary policy instrument with formalised policy reaction functions. Chow, Lim and McNelis (2014) finds that exchange rate rules outperform the Taylor rule when the economy's source of volatility is export-price shocks. Mihov and Santacreu (2013) shows that exchange rate rules surpass Taylor rules when deviations from uncovered interest rate parity are introduced from a time-varying risk premium.

3 Model

This section outlines the non-policy block of my discrete-time, basic New Keynesian model. The model is built from Galí and Monacelli (2005), which uses Calvo (1983) staggered price

[7]See Gopinath, Itskhoki and Rigobon (2010) for an empirical overview of the the LCP vs PCP debate.

setting. I lay the microfoundations of my model and conclude by showing how my model collapses into the canonical three equation representation analogous to a standard closed economy New Keynesian model. It features a forward-looking IS curve and a Phillips curve that depends on expectations of future inflation.

Two focal small open economies (for example Australia and New Zealand), denoted by A and B, are modelled. Country A and B are assumed to be perfectly symmetrical in every way except for their firms' total factor productivity.[8] Since each economy is of measure 0 in the global economy, they both take the rest of the world as a singular, exogenously defined large open economy. Each country is populated by a continuum of identical, infinitely lived agents. There is no migration. Each agent produces a single differentiated good and consumes goods which can be produced in both economies as well as the large country. Analysis will be outlined from country A where, unless otherwise stated, analogous results will also hold for country B.

For notational purposes, variables in levels are denoted with capital letters while variables in lower-case are in logarithmic form. As two focal countries are being modelled, I depart slightly from the notation in Galí and Monacelli (2005) by including a superscript to specify the corresponding country. The large economy is denoted by *.

3.1 Households

A representative household from country A chooses consumption C_t^A and labour supply N_t^A (in hours) to maximise expected discounted lifetime utility:[9]

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U(C_t^A, N_t^A) \tag{1}$$

where $0 < \beta < 1$ is the intertemporal discount factor. The period utility function is assumed to be twice differentiable. I assume that $U_{c,t} \equiv \frac{\partial U_t}{\partial C_t} > 0$, $U_{cc,t} \equiv \frac{\partial^2 U_t}{\partial C_t^2} \leq 0, U_{n,t} \equiv \frac{\partial U_t}{\partial N_t} \leq 0$ and

[8]See Section 3.3.1 for further elaboration.

[9]Following the literature, I assume that money's only explicit rule is to serve as a unit of account.

$U_{nn,t} \equiv \frac{\partial^2 U_t}{\partial N_t^2} \leq 0$. C_t^A is a Dixit and Stiglitz (1977) aggregator of home and foreign bundles of goods defined as:

$$C_t^A \equiv \left[(1-\alpha)^{\frac{1}{\eta}}(C_{A,t}^A)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}(C_{F,t}^A)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{2}$$

where $\alpha \in [0,1]$ is inversely related to the degree of home bias in preferences, $C_{A,t}^A$ is an index of consumption of goods from economy A given by the Constant Elasticity of Substitution (CES) function:

$$C_{A,t}^A \equiv \left(\int_0^1 (C_{A,t}^A(j))^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

where $j \in [0,1]$ denotes the good variety, and $C_{F,t}^A$ is an index of consumption of foreign goods. $\varepsilon > 1$ is the elasticity of substitution across goods produced within any given country and $\eta > 0$ is the elasticity of substitution between $C_{A,t}^A$ and $C_{F,t}^A$. All goods are traded across borders with no trade frictions. $C_{F,t}^A$ is a CES index of goods foreign to country A given by:

$$C_{F,t}^A \equiv \left[(1-\kappa)^{\frac{1}{\eta}}(C_{B,t}^A)^{\frac{\eta-1}{\eta}} + \kappa^{\frac{1}{\eta}}(C_{*,t}^A)^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \tag{3}$$

where $\kappa \in [0,1]$ is inversely related to the degree of country B bias in preferences and $C_{B,t}^A$ and $C_{*,t}^A$ are indexes of consumption goods from country B and the rest of the world, given by the respective CES functions:

$$C_{B,t}^A \equiv \left(\int_0^1 C_{B,t}^A(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}} \quad ; \quad C_{*,t}^A \equiv \left(\int_0^1 C_{*,t}^A(j)^{\frac{\varepsilon-1}{\varepsilon}} dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$$

Note that elasticity of substitution between Country B and the large economy's goods is assumed to be the same as the elasticity of substitution between domestic and foreign goods.

The maximisation of (1) is subject to a sequence of budget constraints for $t = 0, 1, 2, ...,$:

$$\int_0^1 P^A_{A,t}(j)C^A_{A,t}(j)dj + \int_0^1 P^A_{B,t}(j)C^A_{B,t}(j)dj + \int_0^1 P^A_{*,t}(j)C^A_{*,t}(j)dj + \mathbb{E}_t \{\mathcal{Q}_{t,t+1}D_{t+1}\}$$
$$\leq D_t + W^A_t N^A_t + T^A_t \quad (4)$$

where $P^A_{i,t}(j)$ is the price of good j imported from country $i = \{A, B, *\}$ expressed in domestic (country A) currency. W^A_t is the nominal wage per hour, and T^A_t denotes lump-sum transfers or taxes. All previous variables are expressed in units of the domestic currency.

D_{t+1} is the nominal payoff in period $t+1$ of the portfolio held at the end of period t, $\mathcal{Q}_{t,t+1}$ is the stochastic discount factor for one-period ahead nominal payoffs relevant to the domestic households. Households are assumed to have access to a complete set of internationally traded contingent claims, thereby making international financial markets complete.

The optimal allocation of any given expenditure within each category of goods yields the demand functions for country $i = \{A, B, *\}$:

$$C^A_{i,t}(j) = \left(\frac{P^A_{A,t}(j)}{P^A_{i,t}}\right)^{-\varepsilon} C^A_{A,t} \quad (5)$$

where

$$P^A_{i,t} \equiv \left(\int_0^1 P^A_{i,t}(j)^{1-\varepsilon}dj\right)^{\frac{1}{1-\varepsilon}}$$

are the price indices for country A, country B and the rest of the world respectively, expressed in country A's currency.

The optimal allocation of expenditures between foreign goods from country B and the rest of the world is

$$C^A_{B,t} = (1-\kappa)\left(\frac{P^A_{B,t}}{P^A_{F,t}}\right)^{-\eta} C^A_{F,t}; \qquad C^A_{*,t} = \kappa\left(\frac{P^A_{*,t}}{P^A_{F,t}}\right)^{-\eta} C^A_{F,t}$$

and the optimal allocation of expenditures between domestic and foreign goods is

$$C_{A,t}^A = (1-\alpha)\left(\frac{P_{A,t}^A}{P_t^A}\right)^{-\eta} C_t^A; \quad C_{F,t}^A = \alpha\left(\frac{P_{F,t}^A}{P_t^A}\right)^{-\eta} C_t^A$$

Together, the optimal allocation of expenditures on domestic, country B and rest of the world goods respectively is

$$C_{A,t}^A = (1-\alpha)\left(\frac{P_{A,t}^A}{P_t^A}\right)^{-\eta} C_t^A; \quad C_{B,t}^A = \alpha(1-\kappa)\left(\frac{P_{B,t}^A}{P_t^A}\right)^{-\eta} C_t^A; \quad C_{*,t}^A = \alpha\kappa\left(\frac{P_{*,t}^A}{P_t^A}\right)^{-\eta} C_t^A \tag{6}$$

where $P_t^A \equiv \left[(1-\alpha)(P_{A,t}^A)^{1-\eta} + \alpha(P_{F,t}^A)^{1-\eta}\right]^{\frac{1}{1-\eta}}$ is the consumer price index (CPI) for country A and $P_{F,t}^A \equiv \left[(1-\kappa)(P_{B,t}^A)^{1-\eta} + \kappa(P_{*,t}^A)^{1-\eta}\right]^{\frac{1}{1-\eta}}$ is the price index for imported goods to country A in country A's currency.

Hence, total foreign consumption expenditures by households from country A is given by

$$P_{B,t}^A C_{B,t}^A + P_{*,t} C_{*,t}^A = P_{F,t}^A C_{F,t}^A$$

and together, total consumption expenditures by households from country A is given by:

$$P_{A,t}^A C_{A,t}^A + P_{F,t}^A C_{F,t}^A = P_t^A C_t^A$$

This means that (4) can be simplified to:

$$P_t^A C_t^A + \mathbb{E}_t\left\{\mathcal{Q}_{t,t+1} D_{t+1}\right\} \leq D_t + W_t^A N_t^A + T_t^A \tag{7}$$

The following analysis is done with the period isoelastic utility function specification for both countries:

$$U(C_t^A, N_t^A) \equiv \frac{(C_t^A)^{1-\sigma}}{1-\sigma} - \frac{(N_t^A)^{1+\varphi}}{1+\varphi} \tag{8}$$

where $\sigma \geq 0$ is the coefficient of relative risk aversion for households and $\varphi \geq 0$ is the inverse of the elasticity of labour supply.

Assuming perfectly competitive labour markets, a representative household's intratemporal optimality condition is:

$$(C_t^A)^\sigma (N_t^A)^\varphi = \frac{W_t^A}{P_t^A} \tag{9}$$

and the intertemporal optimality condition is:

$$\beta R_t^A \mathbb{E}_t \left\{ \left(\frac{C_{t+1}^A}{C_t^A} \right)^{-\sigma} \left(\frac{P_t^A}{P_{t+1}^A} \right) \right\} = \beta R_t^B \mathbb{E}_t \left\{ \left(\frac{C_{t+1}^B}{C_t^B} \right)^{-\sigma} \left(\frac{P_t^B}{P_{t+1}^B} \right) \right\} = 1 \tag{10}$$

where $R_t^A = \frac{1}{\mathbb{E}_t\{Q_{t,t+1}\}}$ is the gross return on a riskless one-period discount bond paying off 1 unit in domestic currency in period $t+1$.

Log-linearising (9) and (10) around a symmetric steady state yields the following:

$$w_t^A - p_t^A = \sigma c_t^A + \varphi n_t^A \tag{11}$$

$$c_t^A = \mathbb{E}_t \left\{ c_{t+1}^A \right\} - \frac{1}{\sigma}(r_t^A - \mathbb{E}_t \left\{ \pi_{t+1}^A \right\} - \rho) \tag{12}$$

where $\rho = \frac{1}{\beta} - 1$ is the time discount rate, r_t^A is the short term nominal interest rate and $\pi_t^A \equiv p_t^A - p_{t-1}^A$ is CPI inflation.

3.2 International Identities

3.2.1 Bilateral Terms of Trade, Domestic Inflation & CPI Inflation

Country A's **bilateral terms of trade** with country B and the rest of the world are defined respectively as

$$S_{B,t}^A \equiv \frac{P_{B,t}^A}{P_{A,t}^A}; \qquad S_{*,t}^A \equiv \frac{P_{*,t}^A}{P_{A,t}^A}$$

or in logs, $s_{B,t}^A = p_{B,t}^A - p_{A,t}^A$ and $s_{*,t}^A = p_{*,t}^A - p_{A,t}^A$.

The **effective terms of trade** for country A is

$$S_t^A \equiv \frac{P_{F,t}^A}{P_{A,t}^A}$$

or in logs, $s_t^A = p_{F,t}^A - p_{A,t}^A$.

Domestic inflation within country A is the rate of change in the index of domestic goods prices and is given by $\pi_{A,t} \equiv p_{A,t}^A - p_{A,t-1}^A$. Log-linearising the CPI index around a symmetric steady state with $P_A^B = P_A^A = P^A$ yields $p_t^A = (1-\alpha)p_{A,t}^A + \alpha p_{F,t}^A$. This can be combined with a similar log-linear approximation around the symmetric steady state $p_{F,t}^A = (1-\kappa)p_{B,t}^A + \kappa p_{*,t}^A$ to yield

$$p_t^A = (1-\alpha)p_{A,t}^A + \alpha(1-\kappa)p_{B,t}^A + \alpha\kappa p_{*,t}^A \tag{13}$$

Country A's effective terms of trade can be rewritten as $s_t^A = (1-\kappa)p_{B,t}^A + \kappa p_{*,t}^A - p_{A,t}^A$ or expressed as a combination of the terms of trade with country B and that with the large economy:

$$s_t^A = (1-\kappa)s_{B,t}^A + \kappa s_{*,t}^A \tag{14}$$

The effective terms of trade, the price level and domestic prices can therefore be linked in the following relationship:

$$p_t^A = p_{A,t}^A + \alpha s_t^A \tag{15}$$

Combined together, CPI inflation and domestic inflation are linked through:

$$\pi_t^A = \pi_{A,t} + \alpha \Delta s_t^A \tag{16}$$

Hence, the gap between the two measures of inflation is proportional to the percentage change in the effective terms of trade, where the coefficient of proportionality is α, the openness index.

As the **law of one price** is assumed to hold for individual goods for both import and export prices, then for all goods $j \in [0, 1]$

$$P^A_{B,t}(j) = \mathcal{E}^A_{B,t} P^B_{B,t}(j); \qquad P^A_{*,t}(j) = \mathcal{E}^A_{*,t} P^*_{*,t}(j)$$

where $\mathcal{E}^A_{B,t}$ and $\mathcal{E}^A_{*,t}$ is the **bilateral nominal exchange rate**, or the price of country B and the rest of the world's currency respectively, in terms of domestic currency. In logs, $p^A_{B,t} = e^A_{B,t} + p^B_{B,t}$ and $p^A_{*,t} = e^A_{*,t} + p^*_{*,t}$

The **bilateral real exchange rate** between country A and country i is defined as the ratio of the two countries' CPIs, both expressed in terms of domestic currency, as:

$$Q^A_{B,t} \equiv \frac{\mathcal{E}^A_{B,t} P^B_t}{P^A_t}; \qquad Q^A_{*,t} \equiv \frac{\mathcal{E}^A_{*,t} P^*_t}{P^A_t}$$

or in logs, $q^A_{B,t} = e^A_{B,t} + p^B_t - p^A_t$ and $q^A_{*,t} = e^A_{*,t} + p^*_t - p^A_t$.

Country A's real exchange rates can be linked to the terms of trade identities in the following ways which be used later to simplify the goods market clearing condition:

$$q^A_{*,t} = (1 - \alpha\kappa) s^A_{*,t} - \alpha(1 - \kappa) s^A_{B,t} \tag{17}$$

$$q^A_{B,t} = (1 - 2\alpha + \alpha\kappa) s^A_{B,t} \tag{18}$$

Here, it can be seen that the terms of trade's relationship with the real exchange rate decreases as the degree of openness increases.

3.2.2 International Risk Sharing

Assuming complete international financial markets and that the large economy shares country A households' utility function, country B and the rest of the world must also satisfy a condition analogous to country A's consumption Euler equation:

$$\beta \left(\frac{C_{t+1}^B}{C_t^B} \right)^{-\sigma} \left(\frac{P_t^B}{P_{t+1}^B} \right) \left(\frac{\mathcal{E}_{A,t}^B}{\mathcal{E}_{A,t+1}^B} \right) = \mathcal{Q}_{t,t+1}$$

and

$$\beta \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \left(\frac{P_t^*}{P_{t+1}^*} \right) \left(\frac{\mathcal{E}_{A,t}^*}{\mathcal{E}_{A,t+1}^*} \right) = \mathcal{Q}_{t,t+1}$$

where P_t^B and P_t^* respectively are country B and the rest of the world's CPI and $\mathcal{Q}_{t,t+1}$ is the stochastic discount factor. The definition of the real exchange rates and (10) can be combined with the international risk sharing conditions above to result in

$$C_t^A = \vartheta_B^A C_t^B (Q_{B,t}^A)^{\frac{1}{\sigma}} = \vartheta_*^A C_t^* (Q_{*,t}^A)^{\frac{1}{\sigma}}$$

$$C_t^B = \vartheta_A^B C_t^B (Q_{A,t}^B)^{\frac{1}{\sigma}} = \vartheta_*^B C_t^* (Q_{*,t}^B)^{\frac{1}{\sigma}}$$

where $\vartheta_B^A \equiv \mathbb{E}_t \left\{ \frac{c_{t+1}^A}{c_{t+1}^B Q_{B,t+1}^{\frac{1}{\sigma}}} \right\}$, $\vartheta_*^A \equiv \mathbb{E}_t \left\{ \frac{c_{t+1}^A}{c_{t+1}^* Q_{*,t+1}^{\frac{1}{\sigma}}} \right\}$ and $\vartheta_*^B \equiv \mathbb{E}_t \left\{ \frac{c_{t+1}^B}{c_{t+1}^* Q_{*,t+1}^{\frac{1}{\nu}}} \right\}$ represent constants that will depend on initial conditions regarding relative net asset positions. As in Galí and Monacelli (2005), symmetric initial conditions (zero net foreign asset holdings and an ex-ante identical environment) will imply, without loss of generality, that $\vartheta_B^A = \vartheta_*^A = \vartheta_A^B = \vartheta_*^B = \vartheta = 1$.

As the rest of the world is infinitely large, $C_t^* = Y_t^*$ for all t, where Y_t^* is the rest of the world's output and exogenously defined to follow a stationary AR(1) process.

Taking logarithms of the two conditions above and substituting the rest of the world's goods market clearing condition yields:

$$c_t^A = y_t^* + \frac{1}{\sigma} q_{*,t}^A \tag{19}$$
$$= c_t^B + \frac{1}{\sigma} q_{B,t}^A \tag{20}$$

When solving for equilibrium dynamics, it is convenient to express $s_{B,t}^A$ and $s_{*,t}^A$ as a function of exogenous productivity and world output processes (derivation in Appendix B):[10]

$$s_{B,t}^A = \frac{(2+\varphi)\sigma}{\Lambda}(z_t^B - z_t^A) \tag{21}$$

$$s_{*,t}^A = \frac{\varpi}{\Lambda\Omega} z_t^B + \left(\frac{(2+\varphi)\Lambda - \varpi\varphi}{\varphi\Lambda\Omega}\right) z_t^A - \frac{\sigma+\varphi}{\varphi\Omega} y_t^* \tag{22}$$

where z_t^A and z_t^B denotes the total factor productivity of country A and B's firms respectively.

3.2.3 Uncovered Interest Parity

Under complete international financial markets with no arbitrage, differences in interest rates between two countries will equal the relative change in currency foreign exchange rates. Hence, $\frac{1}{R_t^B} = \mathbb{E}_t \left\{ \mathcal{Q}_{t,t+1}(\frac{\mathcal{E}_{B,t+1}^A}{\mathcal{E}_{B,t}^A}) \right\}$ and $\frac{1}{R_t^*} = \mathbb{E}_t \left\{ \mathcal{Q}_{t,t+1}(\frac{\mathcal{E}_{*,t+1}^A}{\mathcal{E}_{*,t}^A}) \right\}$. Since $\frac{1}{R_t^A} = \mathbb{E}_t \left\{ \mathcal{Q}_{t,t+1} \right\}$, the following uncovered interest parity (UIP) conditions are obtained:

$$\mathbb{E}_t \left\{ \mathcal{Q}_{t,t+1} \left[(R_t^A - R_t^* \left(\frac{\mathcal{E}_{*,t+1}^A}{\mathcal{E}_{*,t}^A} \right) \right] \right\} = 0$$
$$\mathbb{E}_t \left\{ \mathcal{Q}_{t,t+1} \left[(R_t^A - R_t^B \left(\frac{\mathcal{E}_{B,t+1}^A}{\mathcal{E}_{B,t}^A} \right) \right] \right\} = 0$$

Log-linearising these conditions around the perfect foresight steady states gives the standard UIP conditions below. Changes in the nominal exchange rate will translate into interest rate

[10]To simplify the notational burden, I introduce the following composite parameters: $\Lambda = \alpha(1-\kappa)[3 - 2(2-\alpha)\eta\sigma - 4\alpha + \alpha\kappa]\varphi - (1-\alpha\kappa)(1-\alpha)\varphi - \sigma$, $\Omega = \left[\frac{[(1-\alpha\kappa)^2 + (2-\alpha)\alpha\kappa\eta\sigma]\varphi + \sigma}{\sigma\varphi}\right]$ and $\varpi \equiv \alpha(1-\kappa)\left[2 - (2-\alpha)\eta\sigma - \alpha\right](2+\varphi)$.

movements through these conditions.

$$r_t^A - r_t^B = \mathbb{E}_t \left\{ \Delta e_{B,t+1}^A \right\}; \qquad r_t^A - r_t^* = \mathbb{E}_t \left\{ \Delta e_{*,t+1}^A \right\}$$

3.2.4 Exports

The demand for country A exports of good $j \in [0,1]$ is given by:

$$X_t^A(j) = \left(\frac{P_{A,t}^A(j)}{P_{A,t}^A} \right)^{-\varepsilon} X_t^A$$

where

$$X_t^A = \alpha(1-\kappa) \left(\frac{P_{A,t}^A}{\mathcal{E}_{B,t}^A P_{B,t}^A} \right)^{-\eta} C_t^B + \alpha\kappa \left(\frac{P_{A,t}^A}{\mathcal{E}_{*,t}^A P_{*,t}^A} \right)^{-\eta} C_t^*$$

Using the global goods market clearing condition $Y_t^* = C_t^*$ and assuming that households' consumption preferences in country A are the same as in country B and in the rest of the world, the above can be rewritten as

$$X_t^A = \alpha(1-\kappa)(S_{B,t}^A)^\eta C_t^B + \alpha\kappa(S_{*,t}^A)^\eta Y_t^*$$

In other words, for each good, the output produced in an economy equals the sum of domestic, country B and rest of the world consumption.

3.3 Firms

3.3.1 Technology

Both economies consists of a continuum of firms indexed by $j \in [0,1]$. Each firm produces a differentiated good using identical technology as represented by the production function:

$$Y_t^A(j) = Z_t^A N_t^A(j) \tag{23}$$

where A_t represents the level of technology, assumed to be common to all firms and evolving according an exogenous AR(1) process: $z_t^A \equiv \log(Z_t^A) = \rho_z^A z_{t-1}^A + \varepsilon_t^a$ with $\rho_z^A \in [0,1]$.

Country A and B are assumed to be perfectly symmetrical in every way except for their total factor productivity AR(1) processes. This deviation will be the cause behind the different results between the two countries.[11]

3.3.2 Price Setting

Prices are set as in Calvo (1983), in which the fraction $1-\theta$, $\theta \in [0,1]$, of randomly selected firms set new prices every period. An individual firm's probability of re-optimising in any given period being independent of the time elapsed since it last reset its price. When reoptimising prices, firms are aware that they may be unable to reset prices for a certain period of time. Domestic inflation therefore depends on future expectations, which generates a forward-looking Phillips curve.

Let τ denote the rate at which the cost of employment is subsidised, which is financed by a lump-sum tax used to offset firms' monopolistic power as explained in Section 4.

As the marginal product of labour is $mpn_t^A = z_t^A$, then if such an employment subsidy is in place, real marginal cost can be expressed as

$$mc_{A,t}^r = -v + w_t^A - p_{A,t}^A - z_t^A \tag{24}$$

where $v \equiv -\ln(1-\tau)$.

The optimal price setting strategy for the typical firm resetting its price in period t can be approximated by the log-linear rule:

$$\bar{p}_{A,t} = \mu + (1-\beta\theta)\sum_{k=0}^{\infty}(\beta\theta)^k \mathbb{E}_t\left\{mc_{A,t+k}^r + p_{A,t}\right\} \tag{25}$$

[11] As can be seen from (21), country A's terms of trade with country B will always be 1 (in levels) under symmetric productivity between countries. See Appendix A of Galí and Monacelli (2005) for a derivation.

where $\bar{p}_{A,t}$ denotes the log of newly set domestic prices, and $\mu \equiv \log(\frac{\varepsilon}{\varepsilon-1})$, which corresponds to the log of the gross mark-up in the steady state, or the flexible price economy.

Domestic inflation within country A is linked to firms' marginal costs via:

$$\pi_{A,t} = \beta \mathbb{E}\left\{\pi_{A,t+1}\right\} + \lambda \hat{mc}^r_{A,t} \tag{26}$$

where $\lambda \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$ and $\hat{mc}^r_{A,t} \equiv mc^r_{A,t} - mc^r_A$ is the markup gap. Thus, the relationship between domestic price inflation and the markup gap is not affected by the degree of openness or the substitutability between domestic and foreign goods in the open economy.

3.4 Equilibrium

3.4.1 Aggregate Demand and Output

The goods market clearing condition is for all $j \in [0,1]$ and for all t:

$$\begin{aligned} Y_t^A(j) &= C^A_{A,t}(j) + X_t^A(j) \\ &= \left(\frac{P^A_{A,t}(j)}{P_{A,t}}\right)^{\varepsilon}\left[(1-\alpha)\left(\frac{P^A_{A,t}}{P^A_t}\right)^{-\eta} C_t^A + \alpha(1-\kappa)(\mathcal{S}^A_{B,t})^{\eta} C_t^B + \alpha\kappa(\mathcal{S}^*_{A,t})^{\eta} Y_t^*\right] \end{aligned}$$

Aggregate domestic output in country A is $Y_t^A \equiv \left(\int_0^1 Y_t^A(j)dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$ which when combined with the condition above results in:

$$Y_t^A = (1-\alpha)\left(\frac{P^A_{A,t}}{P^A_t}\right)^{-\eta} C_t^A + \alpha(1-\kappa)(\mathcal{S}^A_{B,t})^{\eta}\left[C_t^A (Q^A_{B,t})^{-\frac{1}{\sigma}}\right] + \alpha\kappa(\mathcal{S}^A_{*,t})^{\eta} Y_t^*$$

Log-linearising country A's goods market condition around a symmetric steady state and substituting in (18) results in:

$$y_t^A = (1-\alpha\kappa)c_t^A + \frac{\alpha(1-\kappa)}{\sigma}((2-\alpha)\eta\sigma - 1 + 2\alpha - \alpha\kappa)s^A_{B,t} + (2-\alpha)\kappa\alpha\eta s^A_{*,t} + \alpha\kappa y_t^* \tag{27}$$

Combining (16) and the consumption Euler equation gives

$$c_t^A = \mathbb{E}_t\left\{c_{t+1}^A\right\} - \frac{1}{\sigma}(r_t^A - \mathbb{E}_t\left\{\pi_{A,t+1}\right\} - \rho) + \frac{\alpha}{\sigma}\mathbb{E}_t\left\{\Delta s_{t+1}^A\right\} \tag{28}$$

Rearranging the goods market clearing condition for c_t^A, substituting it into (28) and then collecting like-terms yields a version of the Dynamic IS (DIS) curve:[12]

$$y_t^A = \mathbb{E}_t\left\{y_{t+1}^A\right\} + \Phi_A z_t^A - \Theta_A z_t^B - \left(\frac{\sigma + \varphi + \alpha\kappa\varphi\Omega}{\varphi\Omega}\right)\mathbb{E}_t\left\{\Delta y_{t+1}^*\right\} - \frac{1-\alpha\kappa}{\sigma}(r_t^A - \mathbb{E}_t\left\{\pi_{A,t+1}\right\} - \rho) \tag{29}$$

Thus, the DIS equation is similar to the one in a closed economy, except now, there is an additional term linking domestic output to the large economy's output and the country B's productivity. Note that if α and η are high enough, output becomes more sensitive to real rate changes than in the closed economy case. This is because the direct negative effect of an increase in the real rate on aggregate demand and output is amplified by the induced real appreciation and the consequent switch of expenditure towards foreign goods. This will be partly offset by any increase in CPI inflation relative to domestic inflation induced by the expected real depreciation, dampening the change in consumption based on the real rate.

3.4.2 Marginal Cost and Inflation

The labour market clearing condition is

$$N_t^A \equiv \int_0^1 N_t^A(j)dj = \left(\frac{Y_t^A}{S_t}\right)\int_0^1 \left(\frac{P_{A,t}(j)}{P_{A,t}}\right)^{-\varepsilon} dj$$

Log-linearising (23) up to a first order gives

$$n_t^A = y_t^A - z_t^A \tag{30}$$

[12] To lighten the notational burden, the following composite parameters are introduced: $\Phi_A = \left[\frac{\alpha(1-\kappa)[2-(2-\alpha)\eta\sigma-2\alpha](2+\varphi)}{\Lambda} + \frac{\alpha\kappa[1-\alpha\kappa-(2-\alpha)\eta\sigma][\varpi\varphi-(2+\varphi)\Lambda]}{\sigma\varphi\Lambda\Omega}\right](1-\rho_z^A)$ and $\Theta_A = \left[\frac{\alpha(1-\kappa)[2-(2-\alpha)\eta\sigma-2\alpha](2+\varphi)}{\Lambda} + \frac{\alpha\kappa\varpi[1-\alpha\kappa-(2-\alpha)\eta]}{\sigma\Lambda\Omega}\right](1-\rho_z^B)$.

The real marginal cost can be derived from substituting the household intra-optimality condition, international risk sharing condition and (15) into (24):

$$\begin{aligned} mc^r_{A,t} &= -v + w^A_t - p^A_{A,t} - z^A_t \\ &= -v + \sigma y^*_t + \varphi y^A_t + s^A_{*,t} - (1+\varphi) z^A_t \\ &= -v + \left(\frac{\sigma\varphi\Omega - \sigma + \varphi}{\varphi\Omega}\right) y^*_t + \varphi y^A_t + \frac{\varpi}{\Lambda\Omega} z^B_t + \left(\frac{(2+\varphi)\Lambda - \varpi\varphi}{\varphi\Lambda\Omega} - 1 - \varphi\right) z^A_t \end{aligned} \tag{31}$$

Domestic output affects marginal costs through its impact on employment as captured by φ. World output affects marginal costs through its effect on consumption and hence, the real wage as captured by σ. The effect of world output on marginal costs is positive if $\sigma\varphi\Omega + \varphi - \sigma > 0$. This is because with sufficiently high σ, the size of the real appreciation needed to absorb the change in relative supplies is small, with its negative effect on marginal costs offset by the positive effect from a higher real wage.

3.4.3 Equilibrium Dynamics

Since $mc^r = -\mu$, the flexible price equivalent of the above is:

$$mc^r_A = -\mu = -v + \left(\frac{\sigma\varphi\Omega - \sigma + \varphi}{\varphi\Omega}\right) y^*_t + \varphi \bar{y}^A_t + \frac{\varpi}{\Lambda\Omega} z^B_t + \left(\frac{(2+\varphi)\Lambda - \varpi\varphi}{\varphi\Lambda\Omega} - 1 - \varphi\right) z^A_t$$

The natural level of output ($\bar{y}^A_t$) can be solved:

$$\bar{y}^A_t = \frac{v-\mu}{\varphi} + \left(\frac{\sigma - \sigma\varphi\Omega - \varphi}{\varphi^2\Omega}\right) y^*_t - \frac{\varpi}{\Lambda\Omega\varphi} z^B_t + \left(\frac{1+\varphi}{\varphi} + \frac{\varpi\varphi - (2+\varphi)\Lambda}{\varphi^2\Lambda\Omega}\right) z^A_t$$

The effect of world output on natural output is ambiguous, depending on the effect of world output on domestic marginal costs.

Letting $x^A_t \equiv y^A_t - \bar{y}^A_t$ be country A's domestic output gap from the flexible price output,

$$\hat{mc}^r_{A,t} = mc^r_{A,t} - mc^r_A = \varphi x^A_t$$

Substituting this back into (26) means that the following expression for the New Keynesian Phillips curve (NKPC) is obtained:

$$\pi_{A,t} = \beta \mathbb{E}\left\{\pi_{A,t+1}\right\} + \lambda\varphi x_t^A \tag{32}$$

To express the DIS curve in terms of the output gap, note that the real interest rate, using Fisher's identity, is defined as $rr_t^A \equiv r_t^A - \mathbb{E}_t\left\{\pi_{A,t+1}\right\}$. Natural output can be expressed as a function of the natural real interest rate $\bar{rr}_t^A$ and exogenous shocks

$$\bar{y}_t^A = \mathbb{E}_t\left\{\bar{y}_{t+1}^A\right\} + \Phi_A z_t^A - \Theta_A z_t^B + \Psi_A \mathbb{E}_t\left\{\Delta y_{t+1}^*\right\} - \frac{1-\alpha\kappa}{\sigma}(\bar{rr}_t^A - \rho) \tag{33}$$

Note that the degree of openness α influences the sensitivity of output to any given change in the domestic real rate $r_t^A - \mathbb{E}_t\left\{\pi_{A,t+1}\right\}$.

When the above equation is subtracted from the DIS curve earlier, this gives us the DIS curve in terms of the output gap and the natural real interest rate $\bar{rr}_t^A$:

$$x_t^A = \mathbb{E}_t\left\{x_{t+1}^A\right\} - \left(\frac{1-\alpha\kappa}{\sigma}\right)\left(r_t^A - \mathbb{E}_t\left\{\pi_{A,t+1}\right\} - \bar{rr}_t^A\right) \tag{34}$$

The output gap depends on expectations of the future output gap and the deviation of the real interest rate from the natural real interest rate. As shown below, $\bar{rr}_t^A$ is a function of domestic and the other small open economy's technology and expected changes in international output. $\bar{rr}_t^A$ can be extracted from above after substituting in (29) and (33), yielding:

$$\bar{rr}_t^A = \rho + \frac{\sigma}{1-\alpha\kappa}\left(\Psi_A + \frac{\sigma - \sigma\varphi\Omega - \varphi}{\varphi^2\Omega}\right)\mathbb{E}_t\left\{\Delta y_{t+1}^*\right\}$$
$$+\left(\frac{\sigma}{1-\alpha\kappa}\right)\left[\left(\frac{(2+\varphi)\Lambda - \varpi\varphi}{\varphi^2\Lambda\Omega} - \frac{1+\varphi}{\varphi}\right)(1-\rho_z^A) + \Phi_A\right] z_t^A + \left(\frac{\sigma}{1-\alpha\kappa}\right)\left[\left(\frac{\varpi}{\Lambda\Omega\varphi}\right)(1-\rho_z^B) - \Theta_A\right] z_t^B$$

With identical steps, Country B's DIS curve, NKPC and natural real rate can also be solved for, where they respectively are:[13]

$$x_t^B = \mathbb{E}_t\left\{x_{t+1}^B\right\} - \left(\frac{1-\alpha\kappa}{\sigma}\right)\left(r_t^B - \mathbb{E}_t\left\{\pi_{B,t+1}\right\} - \bar{rr}_t^B\right)$$

$$\pi_{B,t} = \beta\mathbb{E}\left\{\pi_{B,t+1}\right\} + \lambda\varphi x_t^B$$

$$\begin{aligned}\bar{rr}_t^B = \frac{\sigma}{1-\alpha\kappa}\left[\Psi_B + \frac{\sigma+\varphi-\sigma\varphi\Omega}{\varphi^2\Omega}\right]\mathbb{E}_t\left\{y_{t+1}^*\right\} + \frac{\sigma}{1-\alpha\kappa}\left[\Phi_B + \frac{(2+\varphi)(\Lambda+\sigma\varphi\Omega)-\varpi\varphi}{\varphi^2\Lambda\Omega}(1-\rho_z^A)\right]z_t^A \\ + \frac{\sigma}{1-\alpha\kappa}\left[\Theta_B + \left(\frac{\varpi-(2+\varphi)\sigma\Omega}{\varphi\Lambda\Omega} - \frac{1+\varphi}{\varphi}\right)(1-\rho_z^B)\right]z_t^B + \rho\end{aligned}$$

It is worth noting that there are three special cases nested within my model. For any non-zero value of α, setting $\kappa = 1$ gives back the original small open economy model in Galí and Monacelli (2005) while setting $\kappa = 0$ implies that country A and B only interact with each other and no other economy in the world. Finally, calibrating $\alpha = 0$ gives back the closed economy basic New Keynesian economy as in Chapter 3 of Galí (2015).

3.5 Rest of the World

The large economy is exogenous to both country A and B and as in Galí and Monacelli (2005), I assume that output follows an AR(1) process:

$$y_t^* = \rho_y y_{t-1}^* + \varepsilon_t^*$$

I also assume constancy of world prices ($\Pi_t^* = 0$), that the goods market clearing condition is $Y_t^* = C_t^*$. Following Mihov and Santacreu (2013), the large economy's nominal interest

[13] To ease the notational burden, I define the following composite parameters $\Gamma = \alpha[(1-\kappa)(1-2\alpha+\alpha\kappa) - (2-\alpha)\eta\sigma + (1-\alpha\kappa)^2]$, $\Phi_B = \left(\frac{\alpha\kappa\left((1-\alpha\kappa)^2-(2-\alpha)\eta\sigma\right)}{(1-\alpha\kappa)\sigma}\right)\left(\frac{\varpi\varphi-(2+\varphi)\Lambda}{\varphi\Lambda\Omega}\right)(1-\rho_z^A) - \left(\frac{\Gamma(2+\varphi)}{\Lambda(1-\alpha\kappa)}\right)(1-\rho_z^A)$, $\Theta_B = \left(\frac{\Gamma(2+\varphi)}{\Lambda(1-\alpha\kappa)}\right)(1-\rho_z^B) - \left(\frac{\alpha\kappa\left((1-\alpha\kappa)^2-(2-\alpha)\eta\sigma\right)}{(1-\alpha\kappa)\sigma}\right)\left(\frac{\varpi}{\Lambda\Omega}\right)(1-\rho_z^B)$ and $\Psi_B = \left(\frac{\alpha\kappa\left((2-\alpha)\eta\sigma-(1-\alpha\kappa)^2\right)}{(1-\alpha\kappa)\sigma}\right)\left(\frac{\sigma+\varphi}{\varphi\Omega}\right) - \alpha\kappa$.

rate is determined by the rest of the world's consumption Euler equation:

$$\beta R_t^* \mathbb{E}_t \left\{ \left(\frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \right\} = 1$$

Hence, the global economy is represented by country A and country B's aggregate demand and supply equations, the risk sharing conditions, the uncovered interest parity conditions, and the law of motion for the exogenous productivity and the large economy's output shocks.

4 Monetary Policy

As in Galí and Monacelli (2005), I assume that an optimal subsidy τ, set such that $(1 - \tau)(1 - \alpha) = 1 - \frac{1}{\varepsilon}$, is in place to offset the firms' monopolistic distortion and the terms of trade distortion.[14] This guarantees the optimality of the flexible price equilibrium allocation.

To close the model, I specify how monetary policy is conducted in Sections 4.1 and 4.2. I consider policies under commitment, where monetary authorities cannot ignore past decisions. Corsetti and Pesenti (2001) shows that under discretion, monetary authorities have incentives to systematically affect the economy through unanticipated changes in monetary policy because of terms of trade externalities. However, monetary authorities under commitment cannot systematically affect the economy through monetary surprises.

I rank the two regimes with a quadratic welfare loss function that represents a second-order Taylor series approximation to the level of expected utility of the representative household in equilibrium with a given monetary policy.

I use the welfare loss function from Galí and Monacelli (2005):[15]

$$W_{A,t} = -\frac{1-\alpha}{2} \sum_{t=0}^{\infty} \beta^t \left[\frac{\varepsilon}{\lambda} \pi_{A,t}^2 + (1+\varphi) x_t^A \right] \tag{35}$$

[14] See Galí and Monacelli (2005) for a more careful derivation of the optimal subsidy in an open economy. Note that in a closed economy basic New Keynesian model, the optimal employment subsidy is $\tau = \frac{1}{\varepsilon}$.

[15] See Section 7 for a discussion of the potential implications of using an ad hoc loss function.

$$W_{B,t} = -\frac{1-\alpha}{2}\sum_{t=0}^{\infty}\beta^t\left[\frac{\varepsilon}{\lambda}\pi_{B,t}^2 + (1+\varphi)x_t^B\right] \tag{36}$$

4.1 Non-cooperative Monetary Policy

Both countries' monetary policy is a managed float exchange rate regime with the large economy, from which the nominal interest rate can be inferred from the UIP conditions. The central bank has a target for the change in its nominal exchange rate with the large economy, $\Delta e^A_{*,t+1}$, based on the state of the domestic economy. Both monetary authorities are assumed to care about stabilising domestic inflation and output. As in Parrado (2004), I specify the following monetary policy reaction function for countries A and B:

$$\Delta e^A_{*,t+1} = \Delta\bar{e}^A_* + \phi_\pi\pi_{A,t} + \phi_x x_t^A \tag{37}$$

$$\Delta e^B_{*,t+1} = \Delta\bar{e}^B_* + \phi_\pi\pi_{B,t} + \phi_x x_t^B \tag{38}$$

where $\Delta\bar{e}^A_*$ and $\Delta\bar{e}^B_*$ is the long-run equilibrium change in the nominal exchange rate with the large economy and ϕ_π and ϕ_x are reaction coefficients that are assumed to be positive.

4.2 Cooperative Monetary Policy

Monetary policy cooperation is modelled by both setting both countries' nominal exchange rate with the rest of the world to respond to both countries' economic conditions:[16]

$$\Delta e^A_{*,t} = \Delta\bar{e}^A_* + \phi_\pi\pi_{A,t} + \phi_x(x_t^A + x_t^B) \tag{39}$$

$$\Delta e^B_{*,t} = \Delta\bar{e}^B_* + \phi_\pi\pi_{B,t} + \phi_x(x_t^B + x_t^A) \tag{40}$$

[16] Although gains to coordination are present if both exchange rules are modified to respond to both countries' domestic inflation instead of the output gap, these gains are more muted. This preserves the optimality of domestic inflation stabilisation from Galí and Monacelli (2005).

5 Calibration

I now calibrate the model and compare the welfare performance of the two rules. I report and analyse impulse responses, second moments, and welfare losses. I use numerical simulations on *Dynare* to generate these results. Time is taken to be quarters and parameter values are summarised in Table 2 in Appendix A.

As the welfare-loss function I use to evaluate various monetary policy regimes' performance is from Galí and Monacelli (2005), I follow their parameter calibration as closely as possible. Hence, $\sigma = \eta = 1$, which is the special configuration used to derive a second-order approximation of the utility of the small open economy's representative consumer.

$\varphi = 3$, which consequently implies a value for the steady-state mark-up $\mu = 1.2$. This implies that $\varepsilon = 6$. θ is set to 0.75, a value consistent with an average period of one year between price adjustments. In keeping with the literature, $\beta = 0.99$, which implies a riskless annual return of about 4% in the steady state. I set the degree of openness, α, to 0.4 which corresponds roughly to the import/GDP ratio in Canada, which the NOEM literature takes as the standard small open economy. κ is set to 0.5, implying no bias between the other small open economy and the large economy. To calibrate the exchange-rate rule, I follow Galí and Monacelli (2005) and set $\phi_\pi = \phi_x = 1.5$ for both countries.[17]

To calibrate the exogenous processes, Galí and Monacelli (2005) uses quarterly HP-filtered data over the 1963:1 - 2002:4 period to estimate AR(1) processes to (log) labour productivity in Canada and (log) U.S. GDP (the proxy for world output):

$$z_t^A = \underset{(0.06)}{0.66} z_{t-1}^A + \varepsilon_t^a, \quad \sigma_{z^A} = 0.0071$$

$$z_t^B = \underset{(0.06)}{0.65} z_{t-1}^B + \varepsilon_t^b, \quad \sigma_{z^B} = 0.0071$$

$$y_t^* = \underset{(0.04)}{0.86} y_{t-1}^* + \varepsilon_t^*, \quad \sigma_{y^*} = 0.0078$$

[17] See Appendix D for welfare loss results under different values of ϕ_π and ϕ_x.

The parameters for Country B's total factor productivity process are chosen to be as close to Country A's as possible. They are only different in their persistence so that any differences between Country A and B can be isolated to this cause.

6 Results

6.1 Impulse Response Functions

I compare the exchange rate regimes by computing impulse response functions to a 1 standard deviation shock in the large economy's output. Figures 1 and 2 show the results. In keeping with the literature, the output gap, terms of trade, the nominal and real exchange rates are expressed in terms of percentage deviations from steady state. Domestic and CPI inflation, nominal and real interest rates are expressed as an annualised rate in percentage form.

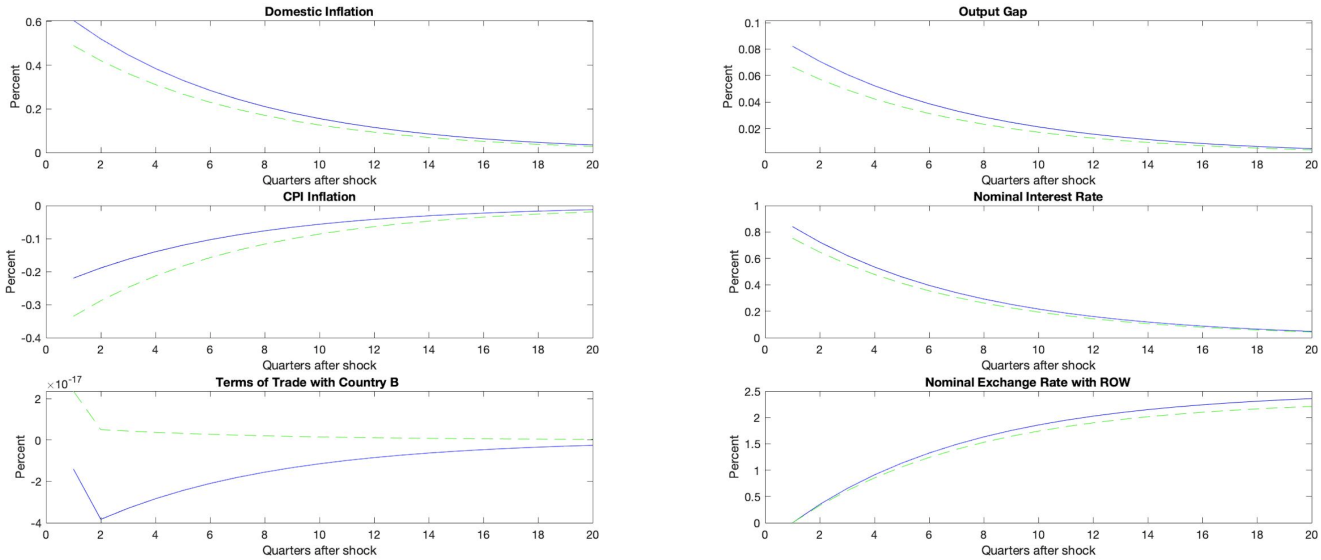

Figure 1: Country A Impulse Response Functions

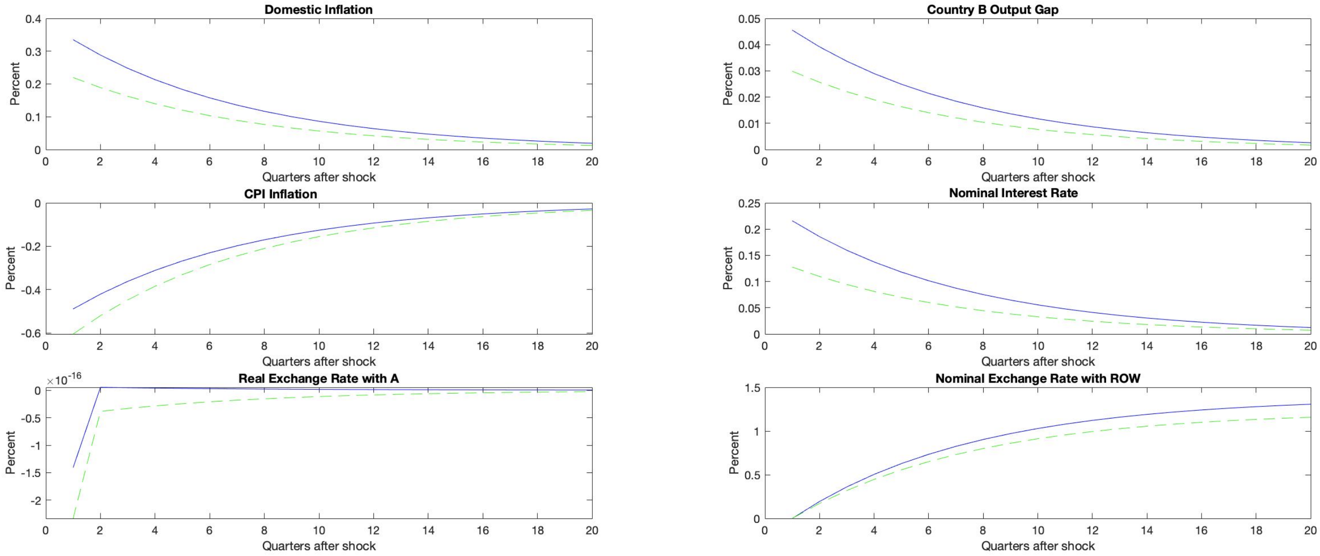

Figure 2: Country B Impulse Response Functions

After a positive output shock from the large economy, there are only small differences in the business cycle dynamics under the two rules. The shock induces an excess demand for small country goods from the large economy, causing the domestic currency to appreciate. Domestic inflation increases and the central bank announces a path of expected appreciation of the currency. However, CPI inflation increases because the announced appreciation does not fully compensate the increase in domestic inflation.

With the exchange rate rule, variables that are affected by exchange rate fluctuation are more stabilised without increasing the volatility of domestic variables. Note that while the terms of trade with the large economy shifts slightly in response to the shock, its difference is almost negligible when considered next to the other variables.

When comparing the two polices, we see that the cooperative exchange rate regime seems to generate lower fluctuations than the non-cooperative exchange rate regime when the economy is hit by shocks from the rest of the world. This suggests that small open economies that cooperative regimes may indeed insulate exchange rate coordinating economies.

6.2 Moments

Table 1 displays the business cycle properties of Country A and B's key variables under the two monetary policies. All results are reported as standard deviations in percentage terms.

Table 1: Cyclical Properties

	Non-cooperative	**Cooperative**
Country A Output	0.014	0.014
Country A Domestic Inflation	0.003	0.003
Country A CPI Inflation	0.008	0.009
Country A Consumption	0.009	0.010
Country A Effective Terms of Trade	0.017	0.017
Country B Output	0.007	0.008
Country B Domestic Inflation	0.002	0.001
Country B CPI Inflation	0.007	0.007
Country B Consumption	0.006	0.006
Country B Effective Terms of Trade	0.010	0.010

As will be shown in Section 6.3, there are minimal differences between the two regimes with differences in many key variables that are too small even at 4 decimal places. As both exchange rate rules respond to domestic inflation instead of CPI inflation, domestic inflation is lower than its CPI equivalent in both countries, although the difference is very minimal. Differences in cyclical properties between country A and B are driven by the minute difference in productivity persistence.

6.3 Welfare Losses

Using Country A and Country B's welfare loss functions, I now compare welfare under the two regimes. All results are given in percentages of steady state consumption.

Under my baseline parameters in Table 2, welfare losses under non-cooperative monetary policy are 0.0268 in country A and 0.0053 in country B. Under cooperative monetary policy, welfare losses are lower at 0.0265 for country A and 0.0022 for country B, indicating positive but small gains to exchange rate coordination. This means that households in country A and B are better off by 0.0003% and 0.0031% of steady state consumption respectively in the cooperative regime.

These results imply that for larger welfare gains from cooperation to exist, additional rigidities and/or different shocks are required. Since the already small gains to cooperation reported in the NOEM literature are the result of the *globally* optimal policy, the lack of more sizeable gains to exogenous definition of exchange rate coordination is not surprising. It is significant however, that benefits to policy coordination are present even under ad hoc policy functional forms that central banks can implement.

Note that while Country B's welfare gains are small they are still larger than Country A's. Given how Country A and B only differ in their total factor productivity persistence with Country A's being higher, this implies that ceteris paribus, higher total factor productivity persistence leads to lower welfare. This effect is closely related to how strongly the monetary authority responds to domestic inflation.[18] When the policy reaction function responds more aggressively to domestic inflation, the real interest rate rises, which creates a counter-effect on inflation, since a higher real interest rate causes the output gap to fall.

As total factor productivity persistence increases, monetary policy's effect on domestic inflation becomes more predictable, implying that the natural interest rate is closer to steady-state and the output gap is less affected by technology shocks. However, Pancrazi and Vukotišc (2011) shows that domestic inflation's response to a technology shock is non-monotonically related to total factor productivity persistence.

Hence, higher total factor productivity persistence means that domestic inflation is more affected by a technology shock, which leads to greater inflation variance and thus welfare losses. Although this is slightly offset by the lower output gap variance, my results suggests that the former welfare decreasing effect is stronger.

[18] As seen in Section 6.4.3, increasing ϕ_π and ϕ_x corresponds to lower welfare losses.

6.4 Sensitivity

I now examine the pattern of coordination gains when key parameters are varied within a realistic range. Key parameters of interest are varied while keeping all other benchmark parameters constant as outlined in Table 2.

6.4.1 Degree of Openness

I assume that degree of openness is 0.4, which is high for most economies.[19] It is therefore interesting to examine how these two rules fare in an average economy. I repeat the analysis in Section 6.3 but now with $\alpha \in [0.1, 0.9]$ in 0.1 increments. Results are given in Tables 4 and 5 in Appendix C.

Holding $\kappa = 0.5$ fixed, welfare losses unambiguously trend downwards as the degree of openness increases in both monetary policy regimes. Given that the exchange rate is the monetary policy, this is not surprising as the more trade dependent a country becomes, the stronger the exchange-rate transmission channel on domestic economic conditions. The gains from coordination peak at $\alpha = 0.1$ with gains of 0.0003% and trend downwards with α, with the lowest gains of 0.0001% for $\alpha = 0.9$.

6.4.2 Small Country Bias in Preferences

I also assume no bias in preferences between the other small open economy and the large economy ($\kappa = 0.5$), meaning that 50% of Country A's foreign goods comes from Country B.[20] This too is likely to be an extreme parameter value for most countries, so I repeat my analysis in Section 6.3 but now with $\kappa \in [0.1, 0.9]$ in increments of 0.1. Results are given in Tables 4 and 5 in Appendix C.

Intuitively, the gains from cooperation would be relatively smaller if the two coordinating

[19] Note that $\alpha = 0$ corresponds to a closed economy.

[20] Note that $\kappa = 0$ means that all foreign goods come from the other small open economy (no trade between the small open economies and the large economy) and $\kappa = 1$ means that all foreign goods come from the rest of the world. That is, there is no trade between the two small open economies.

countries are relatively closed off from each other (smaller values of κ). As the two economies become less interconnected, their economic conditions become less correlated and their policy reaction functions (equations 39 and 40) respond less to domestic conditions. Holding $\alpha = 0.4$ constant, the gains from coordination peak at $\kappa = 0.1$ at 0.0034%, and trends downwards as κ increases, even becoming negative at $\kappa = 0.9$ with welfare losses of 0.0151%.

The presence of welfare losses from policy coordination is somewhat surprising. Although Pappa (2004) also finds that lowering the degree of openness between the coordinating economies lowers the gains from coordination, these gains have nonetheless always been positive. I consider two plausible explanations for this.

Firstly, the welfare loss function I use to generate the results in Table 4s and 5 is from Galí and Monacelli (2005), which was not derived with a second small open economy. Hence, the effect of varying exposure to the other small open economy on domestic welfare is likely to be biased. A micro-founded welfare loss function from my model is required to accurately identify the effect of κ on welfare and assess whether there definitely is welfare loss given my definition of coordination.

Secondly, as alluded to in Section 4.2, the results in the literature are computed as the result of a globally optimal joint welfare maximisation problem, not from an ad hoc policy reaction function. It may be that these welfare losses are the result of the specific way that I have defined policy coordination and that under the globally optimal policy, these welfare losses disappear even under extreme values of κ.

6.4.3 Domestic Inflation and Output Gap Feedback Coefficients

To ensure that the gains from monetary policy coordination are not driven by arbitrary values of feedback coefficients, I also vary the reactivity of the nominal exchange rate to domestic inflation and the output gap. Results are shown in Appendix D. I report values for $\phi_\pi, \phi_x \in [1.1, 1.9]$. To satisfy the Blanchard and Kahn (1980) conditions, ϕ_x and ϕ_π must be greater than 1 and although results for $\phi_x, \phi_\pi \geq 1.9$ are not displayed, the same relationship discussed below is maintained as the coefficients increase.[21]

For every combination of ϕ_π, ϕ_x, I find that the cooperative exchange rate regime outperforms the non-cooperative regime. The more aggressively the nominal exchange rate reacts to domestic inflation and the output gap, the lower the welfare losses. Interestingly, the difference in welfare outcomes (coordination benefits) decreases as the coefficients increases.

7 Limitations & Further Work

7.1 Ad hoc Welfare Loss Function

The loss function that I use to evaluate the two regimes is not micro-founded but is from Galí and Monacelli (2005). Although ad hoc objective functions are still widely used,[22] they are inherently flawed because objectives are endogenous. The underlying structure of the economy is tightly linked to the objectives a policy maker concerned with maximising the welfare of the representative agent should pursue. Using micro-founded loss functions to derive and analyse optimal policy would ensure consistency with the model and overcomes the misleading prescriptions that result from using exogenous loss functions.

While my model is a direct extension of Galí and Monacelli (2005) and simulations are run under the same parameter values for consistency, the presence of a second small open country may nonetheless lead to spurious welfare evaluations.

Alternatively, McCallum and Nelson (1999) suggests that a policy rule should perform well across different models. This is a useful criterion when considering model uncertainty. However, as explained in the introduction, a three-country model is necessary to facilitating the possibility of exchange rate coordination. Since no other model in the NOEM literature features more than two economies, it is not currently possible to evaluate the robustness of my result across different models.

[21] Following Schmitt-Grohé and Uribe (2007), studies on optimal monetary policy have restricted feedback coefficients to $\phi_\pi, \phi_x \leq 3$.

[22] See for example Angeloni, Coenen and Smets (2003) and Adolfson, Laséen, Lindé and Svensson (2011).

7.2 Producer Currency Pricing & Local Currency Pricing

In my analysis, I assume that firms set prices in their own currency, or PCP. Under PCP, changes in the nominal exchange rate automatically translate into a change in the price of imported goods. The exchange rate immediately changes the relative price of imported to local goods, and so plays an important role in achieving efficient outcomes.

However, as detailed in Corsetti, Dedola and Leduc (2010), the PCP assumption is questioned by a strand of the literature that favours LCP, where firms preset prices in domestic currency for the domestic market, and in foreign currency for the market of destination. In other words, exporting firms can price discriminate among markets and/or set prices in the buyers' currencies. A currency could be overvalued if the consumer price level is higher at home than abroad when compared in a common currency, or undervalued if the relative price level is lower at home. Hence, currency misalignment is possible under LCP.

An analysis under LCP would be interesting for two reasons. Firstly, it would provide a more realistic flavour to the model as numerous empirical studies show that there are significant deviations from the law of one price.[23] Secondly, Engel (2011) shows that under LCP, exchange rate misalignment is another key trade-off in addition to inflation and the output gap, pointing to a greater role of the exchange rate. It is possible that the gains from exchange rate coordination will be significantly different under LCP.

8 Conclusion

I study whether there are gains from exchange rate coordination in a novel two-country extension of the small open economy New Keynesian model from Galí and Monacelli (2005).

I find welfare gains to exchange rate coordination, even with a simple exchange rate policy reaction function. Although these gains are robust to parameter values, they peak at only 0.0034% of steady-state consumption. If the degree of openness decreases to values that are

[23] See for example, Alessandria (2004), Crucini and Shintani (2008) and Sarno, Taylor and Chowdhury (2004).

reflective of most countries' openness, then the exchange rate's ability to minimise welfare losses and facilitate gains from coordination diminish rapidly. Furthermore, if the degree of bias in consumption between the other small country and the rest of the world becomes too low, then the non-cooperative policy outperforms the cooperative policy.

Nonetheless, it is significant that gains emerge even from simple definitions of exchange rate coordination that central banks can implement. Although this paper demonstrates that countries would only find benefits from cooperation if they are highly open and trade heavily with each other, perhaps incorporating additional rigidities such as currency misalignment under LCP will yield more cooperative benefits.

I conclude by noting that my model abstracts away from geo-political game-theoretic considerations. I assume monetary policy under commitment from all countries, which eliminates the possibility of countries strategically deviating from coordination in each period. Such possibilities are beyond the scope of this paper.

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Appendix

Appendix A: Parameter Values

Table 2: Parameter Values

Parameter	**Description**	**Value**
β	Discount factor	0.99
ε	Elasticity of substitution between varieties	6.00
φ	Inverse of the elasticity of labour supply	3.00
σ	Country A relative risk aversion	1.00
η	Elasticity of substitution between home and foreign goods	1.00
θ	Probability that a firm will be unable to change its price	0.75
$1-\alpha$	Home - foreign bias in preferences	0.40
$1-\kappa$	Small country - large country bias in preferences	0.50
ϕ_π	Inflation reaction coefficient	1.50
ϕ_x	Output gap reaction coefficient	1.50

Appendix B: Solving for $s^A_{B,t}$ and $s^A_{*,t}$

Firstly, substitute the international risk sharing condition and the large economy's goods market clearing condition into country A's goods market clearing condition:

$$\begin{aligned} y^A_t &= (1-\alpha\kappa)c^A_t + \frac{\alpha(1-\kappa)}{\sigma}((2-\alpha)\eta\sigma - 1 + 2\alpha - \alpha\kappa)s^A_{B,t} + (2-\alpha)\kappa\alpha\eta s^A_{*,t} + \alpha\kappa y^*_t \\ &= y^*_t + \left(\frac{(1-\alpha\kappa)^2 + (2-\alpha)\alpha\kappa\eta\sigma}{\sigma}\right)s^A_{*,t} + \frac{\alpha(1-\kappa)}{\sigma}\left[(2-\alpha)\eta\sigma + 2\alpha - 2\right]s^A_{B,t} \end{aligned}$$

Secondly, households and firms' optimality conditions implies:

$$\begin{aligned}
& mc^r_{A,t} = -v + w^A_t - p^A_{A,t} - z^A_t = mpn^A_t = z^A_t \\
& \implies w^A_t = 2z^A_t + v + p^A_{A,t} \\
& \sigma c^A_t + \varphi n^A_t = w^A_t - p^A_t \\
& \implies w^A_t = \sigma c^A_t + \varphi n^A_t + p^A_t \\
& \implies \sigma c^A_t + \varphi n^A_t + p^A_t = 2z^A_t + v + p^A_{A,t}
\end{aligned}$$

Substituting in the international risk sharing condition and labour market clearing condition and solving for output yields:

$$y^A_t = \frac{v}{\varphi} + \frac{2+\varphi}{\varphi} z^A_t - \frac{\sigma}{\varphi} y^*_t - \frac{1}{\varphi} s^A_{*,t}$$

Equating the goods market clearing condition with the condition above yields:

$$\frac{v}{\varphi} + \frac{2+\varphi}{\varphi} z^A_t - \frac{\sigma+\varphi}{\varphi} y^*_t = \left[\frac{[(1-\alpha\kappa)^2 + (2-\alpha)\alpha\kappa\eta\sigma]\varphi + \sigma}{\sigma\varphi}\right] s^A_{*,t} - \frac{\alpha(1-\kappa)}{\sigma}\left[2 - (2-\alpha)\eta\sigma - \alpha\right] s^A_{B,t} \tag{41}$$

Repeating the steps above for country B results in:

$$\frac{v}{\varphi} + \frac{2+\varphi}{\varphi} z^B_t - \frac{\sigma+\varphi}{\varphi} y^*_t + \frac{1}{\varphi} s^A_{B,t} = \left[\frac{[(1-\alpha\kappa)^2 + (2-\alpha)\alpha\kappa\eta\sigma]\varphi + \sigma}{\sigma\varphi}\right] s^A_{*,t} + \left[\frac{\alpha(1-\kappa)[1-2\alpha-\alpha\kappa-(2-\alpha)\eta\sigma] - (1-\alpha\kappa)(1-\alpha)}{\sigma}\right] s^A_{B,t} \tag{42}$$

Notice that conditional on z^A_t, z^B_t and y^*_t, (40) and (41) equations constitute a system of two equations in $s^A_{B,t}$ and $s^A_{*,t}$ with a unique solution given by:

$$s^A_{B,t} = \frac{(2+\varphi)\sigma}{\Lambda}(z^B_t - z^A_t)$$

and

$$s^A_{*,t} = \frac{\varpi}{\Lambda\Omega} z^B_t + \left(\frac{(2+\varphi)\Lambda - \varpi\varphi}{\varphi\Lambda\Omega}\right) z^A_t - \frac{\sigma+\varphi}{\varphi\Omega} y^*_t$$

Appendix C: Sensitivity Tables

Table 3: Non-cooperative α - κ Sensitivity

κ \ α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0.0304	0.0270	0.0237	0.0203	0.0169	0.0135	0.0101	0.0068	0.0034
0.2	0.0327	0.0291	0.0254	0.0218	0.0182	0.0145	0.0109	0.0073	0.0036
0.3	0.0351	0.0312	0.0273	0.0234	0.0195	0.0156	0.0117	0.0078	0.0039
0.4	0.0376	0.0334	0.0292	0.0250	0.0209	0.0167	0.0125	0.0083	0.0042
0.5	0.0401	0.0357	0.0312	0.0268	0.0223	0.0178	0.0134	0.0089	0.0045
0.6	0.0428	0.0381	0.0333	0.0286	0.0238	0.0190	0.0143	0.0095	0.0048
0.7	0.0457	0.0406	0.0355	0.0304	0.0254	0.0203	0.0152	0.0101	0.0051
0.8	0.0486	0.0432	0.0378	0.0324	0.0270	0.0216	0.0162	0.0108	0.0054
0.9	0.0517	0.0459	0.0402	0.0345	0.0287	0.0230	0.0172	0.0115	0.0057

Table 4: Cooperative α - κ Sensitivity

κ \ α	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	0.0253	0.0225	0.0197	0.0169	0.0141	0.0113	0.0084	0.0056	0.0028
0.2	0.0281	0.0250	0.0219	0.0188	0.0156	0.0125	0.0094	0.0063	0.0031
0.3	0.0314	0.0279	0.0244	0.0209	0.0174	0.0139	0.0105	0.0070	0.0035
0.4	0.0352	0.0313	0.0274	0.0235	0.0196	0.0157	0.0117	0.0078	0.0039
0.5	0.0398	0.0354	0.0310	0.0265	0.0221	0.0177	0.0133	0.0088	0.0044
0.6	0.0455	0.0404	0.0354	0.0303	0.0253	0.0202	0.0152	0.0101	0.0051
0.7	0.0526	0.0468	0.0409	0.0351	0.0292	0.0234	0.0175	0.0117	0.0058
0.8	0.0619	0.0550	0.0481	0.0413	0.0344	0.0275	0.0206	0.0138	0.0069
0.9	0.0744	0.0661	0.0579	0.0496	0.0413	0.0331	0.0248	0.0165	0.0083

Appendix D: Robustness

Although only results for Country A are reported, Country B's results follow identical trends.

Table 5: Non-cooperative ϕ_π - ϕ_x Sensitivity

ϕ_π \ ϕ_x	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
1.1	0.0607	0.0544	0.0490	0.0444	0.0405	0.0370	0.0340	0.0313	0.0289
1.2	0.0526	0.0475	0.0431	0.0393	0.0360	0.0331	0.0305	0.0282	0.0262
1.3	0.0463	0.0421	0.0384	0.0352	0.0324	0.0299	0.0277	0.0257	0.0239
1.4	0.0412	0.0376	0.0345	0.0318	0.0293	0.0272	0.0252	0.0235	0.0220
1.5	0.0370	0.0339	0.0313	0.0289	0.0268	0.0249	0.0232	0.0216	0.0203
1.6	0.0335	0.0308	0.0285	0.0264	0.0246	0.0229	0.0214	0.0200	0.0188
1.7	0.0305	0.0282	0.0261	0.0243	0.0226	0.0212	0.0198	0.0186	0.0175
1.8	0.0379	0.0259	0.0241	0.0224	0.0210	0.0196	0.0184	0.0173	0.0163
1.9	0.0257	0.0239	0.0223	0.0208	0.0195	0.0183	0.0172	0.0162	0.0153

Table 6: Cooperative ϕ_π - ϕ_x Sensitivity

ϕ_π \ ϕ_x	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
1.1	0.0556	0.0499	0.0450	0.0409	0.0372	0.0341	0.0313	0.0289	0.0267
1.2	0.0496	0.0448	0.0407	0.0371	0.0340	0.0312	0.0288	0.0266	0.0247
1.3	0.0447	0.0406	0.0370	0.0339	0.0311	0.0287	0.0266	0.0247	0.0229
1.4	0.0405	0.0369	0.0338	0.0311	0.0287	0.0265	0.0246	0.0229	0.0214
1.5	0.0369	0.0338	0.0311	0.0287	0.0265	0.0246	0.0229	0.0214	0.0200
1.6	0.0338	0.0311	0.0287	0.0266	0.0246	0.0229	0.0214	0.0200	0.0188
1.7	0.0311	0.0287	0.0266	0.0247	0.0230	0.0214	0.0200	0.0188	0.0176
1.8	0.0287	0.0266	0.0247	0.0230	0.0214	0.0201	0.0188	0.0176	0.0166
1.9	0.0266	0.0247	0.0230	0.0215	0.0201	0.0188	0.0177	0.0166	0.0157