A Model of Credit Risk in Interbank Markets with Interest Rate Spreads

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A Model of Credit Risk in Interbank Markets with Interest Rate Spreads

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Abstract
In simple textbook models of monetary policy implementation, central banks inject or withdraw funds from an aggregate stock of reserves and rely on an efficient interbank market to distribute them and determine an overnight interest rate. The term structure of interest rates then feeds these shocks into the real economy through the usual transmission channels of monetary policy. However, during the extraordinary events that have shaken global financial markets in 2007/08 unusually large spikes between overnight rates and unsecured longer term interbank lending rates, such as LIBOR, were observed.

This thesis provides an explanation for this phenomenon by analysing interbank markets as a screening game in which lending banks face uncertainty about recovery values in case of counterparty default. In this setting, lending banks use contracts in the form of maturity and interest rate pairs to separate borrowers with high and low asset values, the former choosing shorter maturities and receiving lower interest rates in return. Such a model offers an alternative derivation of the interbank term structure which is consistent with the events observed during the crisis and which suggests that effective monetary policy implementation is supported by financial institution transparency.

Key Words: Interbank Market, Screening Game, Monetary Policy
JEL Classification: D82, E43, E51
1 Introduction

During the financial turmoil that arose out of the collapse in US mortgage securitisation markets between 2007 and 2008, one aspect of particular concern to monetary policymakers was the uncharacteristically large spreads between overnight cash rates and longer maturity unsecured rates, such as LIBOR. Traditionally such spreads were stable, which allowed central banks to affect these interbank money market rates by changing overnight rates in a reasonably predictable manner. This short end of the term structure of interest rates therefore plays an important role in the transmission of monetary policy.

As a result, the emergence of such uncharacteristic and volatile spreads in rates made the implementation of monetary policy stimuli substantially more difficult precisely when the collapse in asset prices heralded a contraction in the real economy.

This paper seeks to present a novel approach to explaining the relationship between such spreads and uncertainty about asset values held on bank balance sheets. It is proposed that when banks make loans in interbank markets, they design such loans in order to counter credit risk by offering specific interest rate and maturity combinations to specific counterparty types. In generating such a set, or “menu”, of contracts, they induce borrowing banks to reveal their type and can therefore to some extent overcome the information asymmetry caused by discontinuous financial reporting.

Section 2 will introduce the economic theory behind this mechanism and outline its application to this problem, which is novel to the literature. Section 3 will then briefly outline the key features of the model, followed by a summary of the results presented in Section 4. Section 5 concludes.

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2 Methodology

2.1 Banks and Interbank Markets

Banks are a particular type of economic entity, characterised by the provision of maturity transformation to other agents within the economy. This brings with it a particular kind of risk to which any bank is necessarily exposed. This risk consists of the different maturity profiles of the assets and liabilities in its portfolio, since it is usually not possible to match any sudden outflows from deposits with inflows from a portfolio of loans.

The management of this phenomenon is a major part of a bank’s business. In order to meet expected withdrawals, some proportion of total deposits is kept in the form of reserves (for example as deposits with central banks) and a stock of liquid short term assets is generally kept on the balance sheet. Since reserves generally earn a much lower return than funds invested in other financial assets or loans, there exists a trade-off between safety and profitability.

Reserves are also used by banks to settle transactions between each other as part of normal business. When a depositor of one bank withdraws funds and uses them in a transaction, a different individual will ultimately deposit them with a (potentially) different bank. This requires a transfer of reserve funds from one bank to another. This type of transfer, of which millions occur on any given day, will affect the reserve holdings of individual banks, but would not affect the aggregate stock of reserves. Apart from transactions between the Treasury and the central bank, there are only two forces which may change this aggregate. The first is obvious: if the depositor withdrew the funds and held them in the form of cash without depositing them, there would be a debit from his banks’ reserve account, but no credit to any other bank. The total stock would thus have decreased. Of course, the converse process also occurs on a daily basis.

However, the total stock of reserves does not generally change a great deal, since the popular requirement for physical cash is relatively stable. The other force which may change reserve holding, namely the central bank itself, may be more important in that sense. Modern central banks have three different means of injecting or withdrawing reserves from the system. By far the most commonly
used are their open market operations, which are in effect auctions aimed at buying or selling reserve funds in order to change the aggregate.

When a bank finds itself in the difficult position of facing a liquidity shock large enough to prompt an immediate requirement for additional reserves, it may also access discount lending facilities. Since these programs charge interest some margin above policy rates and need for them signals some failure in liquidity management, these emergency facilities are used more rarely.

It should be noted then that the interbank market in which banks trade reserve funds do not affect the aggregate stock of reserves. Instead, they can only serve to help banks in their individual reserve management. This implies that there are two sides to any trade in this market. On one hand, those banks which believe that their depositor’s liquidity needs will be low for a given period may choose to reduce their holding of reserves without fully committing to lending the funds as illiquid loans into the real economy. On the other, should a bank’s estimation turn out to be incorrect, the difference can be made up quickly within the market by borrowing or lending funds and thus balancing the books.

Interbank Markets are then credit markets, in structure somewhat similar to any other such market. Although they are of particular importance to the functioning of the monetary system, in principle lending banks face credit risks not dissimilar to those they face when lending to other types of agents in the economy. Although such risks were previously considered to be very low, the financial crisis of 2007/2008 highlighted that their management can be critical to the funding strategy and indeed survival of a large financial institution.

2.2 Screening Games

Game Theory and Information Economics are two particularly important fields within microeconomic theory. Simply put, the former concerns itself with the behaviour of rational agents when the pay-offs of their actions depend on the actions taken by others. As can be imagined, this field can cover a vast array of possible situations, from chess to nuclear warfare and even relationships. Information Economics on the other hand arose from the realisation that information imperfections can have a significant impact on the optimal behaviour of economic agents, and therefore the outcomes that can be expected from an
economic situation. Famously, Akerlof (1970) showed that even small uncertainties about the quality of the product for sale can quickly lead to entire markets shifting drastically.

In a similar vein, Stiglitz & Weiss (1981) showed that in credit markets, in which borrowers know more about themselves than lenders do, such asymmetric information leads to a failure of the market to clear. This form of (Type II) rationing represents a breakdown of the standard market mechanism as an efficient allocator of capital and liquidity. In effect, because raising interest rates above a certain level only attracts borrowers with low probabilities of repayment, lenders cannot profitably do so, and thus a profit maximising level of interest rate is one at which there is excess demand for loans.

Milde & Riley (1986) relax the assumptions of this model and allows for the introduction of screening by means of an additional contract dimension. Screening games are defined within Stiglitz & Weiss (1990) as games of asymmetric information in which the uninformed side makes the first move, distinguishing them from signalling games. In effect, this uninformed first mover must take an action designed to extract information about the other player’s type by observing their reaction. Within Milde & Riley (1986), the introduction of loan size as a characteristic of the credit contract allows the (uninformed) lenders to offer a menu of contracts in the form of a vector of loan size and interest rate pairs. Because borrower types have differing marginal rates of substitution of these two dimensions, this potentially allows for the creation of one contract that appeals exclusively to one type of borrower. Thus, the menu offered serves as a screening mechanism that reveals borrower types. If appropriately designed, this can be done profitably by the lenders, leading to an equilibrium outcome without credit rationing.

Of course, there exist a variety of conditions which are necessary in order to allow profitable screening. Chief among these is “Spence-Mirrlees Condition”, as outlined for example in Basov (2005). In effect this condition specifies that the marginal rate of substitution between the relevant contract dimensions is either monotonically increasing or monotonically decreasing with borrower type. Were this not the case, it would not be possible to reliably separate borrower types by means of these two contract dimensions, and hence a screening equilibrium typically does not exist.
2.3 Recent Theoretical Developments in the Analysis of Interbank Term Structures

Perhaps the closest paper in intention to the proposed thesis is being developed by Heider, Hoerova & Holthausen (2009), who present a model of an interbank market under adverse selection. In keeping with much of the modelling of banks, some proportion of the market in their model suffers a liquidity shock as impatient consumers withdraw their endowments. Banks must decide whether to invest those endowments in a liquid short-term or asset or a risky, illiquid long-term alternative. When they face withdrawals, they must then choose whether to liquidate some of their long-term assets or keep them to maturity and instead secure funding for the meantime from other banks. Due to the difference in riskiness between long-term assets (the precise knowledge of which is private), some banks will be willing to lend, while others become borrowers.

Since those banks with a safer asset are assumed to be able to liquidate it at a lower cost, they are the first ones to do so should the interbank interest rate increase beyond a critical value. This gives rise to an adverse selection problem. The authors derive a risk premium, which increases unequivocally as adverse selection becomes a bigger problem, meaning when the expected riskiness of long-term assets increases. Indeed, the authors show that it is possible that the interbank market breaks down entirely, given sufficiently risky assets.

Although this analysis is close in spirit to the one proposed here, its approach is fundamentally different. In Heider et al.’s (2009) paper, interbank markets are defined to have only one maturity. By using the methodology of game theory, it is proposed in this thesis that asymmetric information and increased perceived riskiness does in fact have an impact across the entire term structure of contracts traded between banks.

One of the more recent applications of screening methods is presented by Acharya & Viswanathan (2008), who introduce asset pricing in their bidimensional loan contract model in which firms pledge collateral to counteract a moral hazard situation in a credit market. Since the authors introduce a delay in asset liquidation when firms fail to repay loans, liquidity shocks can have a contagion effect in their model. As in Bester (1987), credit rationing would occur in a pooling equilibrium and as a result, pledging cash collateral is an optimal strategy, requiring sales of assets by the borrower. By modelling a market for asset sales,
the authors establish that by posting collateral, not only is rationing weakened, but asset prices are actually more stable than would have been the case otherwise. The relationship between asset pricing and credit markets is obviously a direct link with the proposed research.

Another model of immediate significance to the research question is presented in Freixas & Jorge (2008), which focuses on modelling interbank markets. In particular it makes use of a screening game to explain why a lending channel of monetary transmission may exist. As such it is a model of the interbank market at the core of the broader term structure. In their model, Freixas & Jorge demonstrate a situation in which banks in interbank markets can be rationed, and thus are unable to provide funds to positive NPV projects in the real economy. The authors produce this result by referring to a chance that banks suffering large liquidity shocks may borrow in order to finance a private benefits project. In the ensuing screening game (in which contracts are defined in terms of an interest rate and loan size), borrowing banks that are undertaking this gamble want to maximise the loan they take out. Indeed it is shown that there exists a loan size above which only such “bad banks” would be attracted and lenders would thus have an incentive to decrease the size, resulting in Type I credit rationing.

At this equilibrium level, interbank interest rates are higher than they would have been otherwise and higher than the official interest rate (which in this model is set exogenously by an external authority). Furthermore, some banks find that they are unable to secure loans and as a consequence the projects undertaken by “their” firms would not receive financing to cover cost overruns.

The model thus has the following two interesting results. Firstly, banks with greater liquidity (thus suffering less of a negative shock) react less strongly to monetary policy in the form of a change in policy rates, a result known as the “Kashyap & Stein Liquidity Effect”. Secondly, where there exists asymmetric information, the model provides support for the lending channel of monetary policy, since credit rationing implies that some profitable investments cannot receive funding at any price.
2.4 A Brief Overview of the Methodology

In the model presented in this paper, banks suffer idiosyncratic liquidity shocks which prompt some banks to become lenders and others to become borrowers in the interbank market. These banks may either approach the central bank to borrow or deposit reserves at some exogenous interest rate over a predetermined time period \( t \), or may trade loan contracts with one another. Should a borrowing bank default while a loan is outstanding, it is liquidated and its assets will be sold in order to restore as much as possible of the borrowed funds to the lender.

The fact that the value of these assets is private information is the source of information asymmetry, and draws directly on the experiences made during the financial turmoil. Off-balance sheet obligations, difficult to value “toxic” assets and modifications and relaxations of accounting standards all contributed to less than perfect transparency within interbank markets. In order to overcome this asymmetric information environment, borrowing banks make use of interest rate and loan maturity as the two contract dimensions. This approach is novel in the literature, since maturity has rarely been used as a screening tool in games of this type.

The screening equilibrium menu can then be defined, and thus specifies a series of interest rate and maturity combinations. In effect, this implies a yield curve and thus a precise specification of the term structure in interbank markets, ranging from the longest possible maturities to the extremely short, overnight, market.

3 The Model

3.1 General Setting

Consider a continuous time setting lasting from \( t = 0 \) to \( t = T \) in which there exist \( N \) banks. At \( t = 0 \), their balance sheets consist of deposits \( (D_i) \) on the liabilities side and reserves \( (R_i) \) and “uncertain assets” \( (A_i) \) on the assets side, where \( i = 1, 2, ..., N \). The value of \( A_i \) is considered private information. This

\[ \text{One may think of a reserve maintenance period as is common in many economies around the world.} \]
is the source of information asymmetry in this model: borrowers will know the size of the assets they hold, while lenders only know the range of possible value realisations. This captures the feature that during the recent crisis financial institutions were more aware of the amount of “toxic assets” they had on their balance sheet and in off-balance sheet entities than the market in general.

Since banks face the possibility of deposits being withdrawn, they hold some proportion of deposits, \( \theta D \), in reserves, where \( 0 < \theta < 1 \). In order to motivate an interbank market, let \( k \) banks suffer a negative exogenous change to deposits of size \( S \). Since they pay the withdrawn funds out of reserves, they now hold \( \theta D - S < \theta(D - S) \) in reserves and must acquire an additional \((1 - \theta)S\) in order to restore the stock to the desired level. This quantity will be equal to \( L \), since all \( k \) borrowing banks suffer the same shock. The banks must secure the reserves for the entire period under consideration, for which they may access either the central bank or an interbank market, characterised by competitive lenders and bi-dimensional loan contracts.

Interbank loans will take the form of simple loans by which both principal and interest fall due at maturity \( \tau \) in a one-off payment. Short loans with a maturity \( \tau < T \) must be rolled over until \( T \). Assume that the roll over is with the same lending bank and the same lending conditions in order to ensure a static game where banks offer a menu of contracts only once at \( t = 0 \). At every roll-over, there is a transaction cost equal to \( \alpha L \) to be paid by the borrower. Furthermore, the interest accrued on the loan will be repaid out of the bank’s income (perhaps generated by the bank’s assets \( A_i \)), such that the amount to be borrowed is always \( L \) at each roll over until \( T - \tau \).

As a result, loan contracts of a constant size \( L \) will be specified as a combination of an interest rate \( (r) \) and a maturity \( (\tau) \), or equivalently, interest rate and the number of roll overs \( (n) \), where \( n = \frac{T}{\tau} \). By requiring that \( n \in \mathbb{N} \), it is in effect assumed that loans cannot be interrupted by the advent of time \( T \). The interest outstanding will then be calculated under a continuous compounding framework, such that the contractual repayment at maturity \( \tau \) will be \( e^{r\tau}L \), of

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2 The withdrawn funds will be deposited with what will become lending banks.
3 In countries with reserve requirements, there would be a given reserve maintenance period, over which the legislated reserve ratio must be maintained and which could determine \( T \).
4 Note well the number of roll overs is equivalent to the number of loans during \( T \), such that when \( n = 1 \), there is one “roll over” equivalent to one loan of maturity \( T \).
which \((e^{\tau} - 1)L\) will be the interest component.\(^5\)

In addition, there exists a central bank, which does not act strategically and which pays interest on reserves deposited \((r)\) and offers a discount window facility which banks may access and which charges an interest rate \((\tau)\). For simplicity, this discount loan and deposit facility will be for a single maturity \(\tau = T\). These interest rates are common knowledge.

### 3.2 Borrowers

How do borrowers value such loan contracts? In order to answer this question, note that the borrowing bank seeks to minimise the expected cost associated with securing \(L\) in reserves from \(t = 0\) until \(T\). At each roll over date there is some exogenous probability \(p\) that it will not default and pay as contractually obliged. In this model, default probabilities are assumed to be independent of asset size, such that default is an exogenous event shock here.\(^6\)

When a bank defaults and the roll over is therefore interrupted, the borrower will expect to suffer some damage, \(D(A)\), the size of which is related to its asset holdings. This damage would not necessarily have to be financial, but could also be losses of reputation, legal action or similar consequences of default. In order to reflect the now common perception that larger banks are more likely to be judged “too big to fail” and hence be bailed out by government help, such that shareholders and managers can expect to suffer less damage in case of default, let \(\frac{\partial D}{\partial A} < 0\). Each bank knows the value of its own asset holding, while other banks merely know the characteristics of the distribution \([A, \bar{A}]\).

If the bank does not default, it will roll over the debt, pay the transaction cost and face the same probabilities of repaying at the next maturity date.

Consider Figure 1 which illustrates this process in the case of a contract that specifies a maturity of \(\tau = \frac{T}{3}\), such that \(n = 3\).

Recalling that a cost of \(\alpha L\) falls due at each roll over and at \(t = 0\), that the probability of defaulting is equal to \(1 - p\), and that the interest component of

\(^5\)Note that \(\lim_{n \to \infty} (1 + \frac{r}{\tau})^{n\tau} = e^{\tau}\) (Hull 2008)

\(^6\)In effect, it is treated as though default is only possible at roll over, with the length of time in between roll overs only important in the sense that it determines how often default could occur.
the repayment is equal to $e^{r\tau}L - L = (e^{r\tau} - 1)L = (e^{rT} - 1)L$, it becomes clear that the overall expected repayment $C$ is a function of the interest rate, number of roll overs and asset values and must be given by

$$C(r, 3, A) = \alpha L + p^2 \left[ 2(e^{rT} - 1 + \alpha)L + e^{rT}L \right] + p^2(1 - p) \left[ 2(e^{rT} - 1 + \alpha)L + D(A) \right] + p(1 - p) \left[ (e^{rT} - 1 + \alpha)L + D(A) \right] + (1 - p)D(A)$$

It should be noted that here, for simplicity of exposition, the subjective discount rates of the borrowing bank is equal zero. This assumption will be maintained throughout the following analysis for both lending and borrowing banks. Alternatively, consider all variables in present values.

When generalising this cost function, due care must be taken that $n$ is necessarily an integer, that is $n \in \mathbb{N}$. Keeping this in mind, Appendix I demonstrates that the generalised version of this cost function for any $n$ roll overs is equal to

$$C(r, n, A) = \frac{p^n - 1}{p - 1} \alpha L + \frac{p(p^n - 1)}{p - 1} e^{rT}L - \frac{p^n - p}{p - 1}L - (p^n - 1)D(A) \quad (1)$$
The isocost schedule of a given borrower type $A$ is then

$$s_h(n, A) = \frac{n}{T} \ln \left[ \frac{p - 1}{p(p^n - 1)} L + \frac{p - 1}{p} \frac{D(A)}{L} + \frac{p^n - p}{p(p^n - 1)} - \frac{\alpha}{p} \right]$$  \hspace{1cm} (2)$$

This function is strictly increasing in $r$, but the effect of $n$ is ambiguous.

A discrete version of the Spence-Mirrlees Condition holds for this setting. As a result, since the marginal rates of substitution between interest rate and number of roll overs changes with borrower asset quality, it is possible to screen between them using those dimensions.

Finally, it is possible that borrowers find the expected cost of receiving a loan from the discount facility provided by the central bank is lower than that of any contract they could secure in the interbank market. This leads to the participation constraint:

$$C \leq \alpha L + (1 - p)D(A) + p^T L$$  \hspace{1cm} (3)$$

### 3.3 Lenders

Once borrowers accept a contract, the lender expects to earn interest payments at each roll over time and full repayment of the principal at $T$. The transaction costs associated with the contracts are not part of the lender’s receipts.

In case of default, the borrowing bank’s balance sheet will be liquidated in order to repay as much as is possible. From the lender’s perspective, borrowing bank assets hence give rise to a recovery value $V(A)$ which stands for the amount the borrower can pay after her assets have been liquidated. In order to ensure an adverse effect of premature default on the lender, it must be the case that $V(A) < e^{rT} L$. This can be guaranteed by assuming that $V(A) < L \forall A$.

Since the value recovered would increase with the size of the asset holdings on the balance sheet, let $\frac{\partial V}{\partial A} > 0$. Given this, one can specify a relationship between the recovery value to the lender and the damage suffered by defaulting borrowers. One would generally expect that this damage would include any sums recovered by creditors, and further include the damage to reputation or possible legal ramifications for decision makers. There is some intuitive justification then for requiring that $D(A) > V(A) \forall A$. 

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Given the borrower characteristics described above, and holding that transaction costs will not be a part of the lender’s income and that the opportunity cost is equal to the return on a deposit with the central bank \((e^T L)\), the expected profit \(\pi\) on an interbank loan is equal to

\[
\pi(r, n, A) = \frac{p(p^n - 1)}{p - 1} e^{\pi r} L - \frac{p^n - p}{p - 1} L - (p^n - 1)V(A) - e^{\pi T} L \quad (4)
\]

Profits are unambiguously increasing in \(r\) and unambiguously decreasing in \(n\). Note well that it is assumed in the following analysis that lenders compete for the entire interbank market, such that they would be able to serve the entire market if necessary.\(^7\)

The zero-profit menu of interest rates is one that yields zero economic profit to the lender for any borrower type. It is given by

\[
r_0 = \frac{n}{T} \ln \left[ \frac{p - 1}{p(p^n - 1)} e^{\pi T} + \left( \frac{p - 1}{p} \right) V(A) + \frac{p^n - p}{p(p^n - 1)} \right] \quad (5)
\]

implying a range of \([r_0(A), r_0(A)]\). Note that its slope is defined as

\[
-\frac{\Delta \pi/\Delta n}{\partial \pi/\partial r}
\]

and is therefore always increasing for any borrower type \(A\). Intuitively, since the number of roll overs \(n\) has an adverse effect on profits, the corresponding interest rate charged must be higher to allow the lender to break even. It should be noted that the slope of the isocost schedule is always less than that of the isoprofit schedule for any \(A\).

\(^7\)This is a simplifying assumption, since the introduction of quantity restrictions would require a different form of partial equilibrium analysis.
3.4 The Perfect Information Equilibrium

The above specification makes it possible to define the perfect information equilibrium as follows:

Figure 2: The Equilibrium Menu of Contracts under Full Information with Two Types

Figure 2 illustrates the perfect information equilibrium for two types of borrowers. The lenders’ zero-profit conditions for both types of borrowers are monotonically increasing functions, reflecting the fact that as $n$ increases, the probability of default during $T$ rises, necessitating a higher interest rate to compensate. The zero-profit condition for the worse borrower, $r_0(n, A)$, is shown in red, while $r_0(n, \overline{A})$ is shown in blue. In the graphed scenario, the isocost curves for both types of lenders are monotonically decreasing functions, with $s_k(n, A)$ shown in purple and $s_k(n, \overline{A})$ shown in green. The implication of this is clear: as $n$ increases, the likelihood of defaulting and suffering $D(A)$ as well as the total transaction cost paid increases, necessitating a lower interest rate in order to generate the same overall expected cost.

Note that the vertical lines in the diagram denote integer values on the $n$-axis, and as such only those points on the schedules should be considered. Represent-
ing the curves as being continuous helps the visualisation substantially however, which is why this presentation is chosen.

No \( n \) larger than one (that is no maturity shorter than \( T \)) will be part of the full information equilibrium menu. It is obvious from the diagram why this is the case. In this competitive market, the interest rate is driven to the zero-profit level. As \( n \) increases, the interest rate required by lenders to break even rises, whereas the interest rate must fall for the borrower to remain on the same isocost curve. Any contract on the zero-profit condition for either lender is associated with a higher expected cost than that of the corner solution at \( n = 1 \).

**Proposition 1.** In a full information environment, where \( D(A) \geq V(A) \), all lending banks offer an identical menu of contracts \([r^*, n^*]\), characterised by

\[
\begin{align*}
    r^*(A) &= \frac{1}{T} \ln \left[ \frac{1}{p} e^{-T} + \frac{p - 1}{p} \frac{V(A)}{L} \right] \\
    n^* &= 1
\end{align*}
\]

(6) (7)

### 3.5 Break-Down of Full Information Equilibrium

When information asymmetry is introduced, this equilibrium breaks down. Since every borrower types chooses the contract designed for the best type, which earns a negative profit when chosen by any borrower worse than \( \bar{A} \), this contract is removed from the menu offered. This process ensures that only the contract offered to the worst borrower type is guaranteed non-negative profits under asymmetric information.

**Proposition 2.** Under asymmetric information, the full information menu is no longer the equilibrium. Furthermore, only the worst borrower is offered her full information menu contract, \((r^*(A), 1)\).

### 3.6 The Screening Equilibrium

At this point it should be examined exactly what screening involves. A strategy profile of a game is considered a Nash equilibrium iff “no player could increase his expected payoff by unilaterally deviating from the prediction of the profile.”
(Myerson 1991). What would it take for this general notion to hold in the context of this screening game?

Firstly, it must not be possible for any of the lenders to offer an alternative menu of contracts which attracts all borrowers of a given type to it. This implies that, since for any number of roll overs $n$ both the expected cost to the borrower and the expected repayment to the lender are decreasing in $r$, the interest rate must be the lowest consistent with non-negative profits for that combination of roll overs and borrower type. Were it possible for a competing lender to offer a marginally lower interest rate, he could profitably attract that entire segment of the market. It should be noted at this stage that the discontinuous nature of one of the contract dimensions here, namely $n$, may lead to the minimum non-negative profit larger than zero.

Secondly, it must not be possible for any type of borrower to choose a contract from the menu designed for another type and reduce their expected cost. This condition is known as the incentive compatibility constraint. Intuitively, it must be the case that the contracts in the equilibrium menu are designed such that all types of borrowers will maximise their payoff by choosing the contract designated for them. The perfect information menu of contracts is an example of incentive compatibility being violated: even though $(r^*(A), 1)$ was designed for $A$, that type of borrower preferred $(r^*(\overline{A}), 1)$ instead. Since that borrower type could naturally be excluded from this contract under perfect information, this was not a problem. Under asymmetric information however, lenders have no choice but to rely on borrowers’ incentive structures to induce them to self-select.

It becomes clear that the Spence-Mirrlees Condition guarantees the possibility of screening for models like this. Since the marginal rate of substitution between the two contract dimensions differs between types in the way specified by the condition, there must exist sets of contracts with are incentive compatible.
A screening equilibrium can then be defined as follows:

**Definition 3.** A menu of contracts \( \{(\hat{r}(A_i), \hat{n}(A_i)), (\hat{r}(A_j), \hat{n}(A_j))\} \) is considered a candidate for asymmetric information equilibrium iff:

\[
\pi(\hat{r}(A_i), \hat{n}(A_i), A_i) \geq 0 \quad (8)
\]
\[
C(\hat{r}(A_i), \hat{n}(A_i), A_i) \leq C(\hat{r}(A_j), \hat{n}(A_j), A_i) \quad (9)
\]
\[\forall A_i, A_j \in [A, \overline{A}]\]

Begin by considering Figure 3:

![Figure 3: Asymmetric Information Equilibrium with Screening](image)

In this graph, the full information equilibrium depicted in Figure 2 has broken down. At \( n = 1 \), the only contract offered is \( (\hat{r}^+(A), 1) \) and hence the isocost schedule \( s_h \) of the type \( A \) (shown in green) is that with the cost equal to that of taking this contract, \( C(\hat{r}^+(A), 1, \overline{A}) \). Note that there is a set of contracts
that fall below that isocost schedule, but above that of the low asset borrower, which is shown in purple. In effect, all of these combinations of interest rate and number of roll overs are such that their cost to A would be higher than that of his full information contract, \( C(r^*(A), 1, A) \), but the cost to \( \bar{A} \) would be lower than \( C(r^*(A), 1, \bar{A}) \). In other words, offering these contracts would attract only \( \bar{A} \) and hence they are incentive compatible.

However, not all of these contracts comply with the non-negative profit condition specified in Definition 3. Indeed, any contract below the zero-profit schedule for the borrower type \( A \), depicted here in blue, would yield a negative profit to the lender if it were offered to that type. Therefore, the only set of contracts which meet Definition 3 are those in the shaded area.

Competition must then drive lenders to offer the most attractive contract in the shaded area to borrowers of type \( \bar{A} \). Where \( n \in \mathbb{N} \), this contract is denoted \( E = (r_E, n_E) \) in the Figure. Any interest below that offered at \( E \) would yield a negative profit, as would any number of roll overs larger than the one associated with that point. Of all incentive compatible, non-negative profit contracts, it minimises borrower cost. Its precise specification is derived in Appendix II.

It is of course not guaranteed that \( E \) is actually achievable itself, given that \( n \) is discrete. When \( E \) falls between integers, lending banks will be required to choose a contract with \( n \) equal to one of the integers around \( n_E \). Denote these two possible contracts \( E' \) and \( E'' \) respectively as in Figure 3. Then it is possible to specify the screening equilibrium as follows:
Proposition 4. The Equilibrium Screening Menu in a market with two types of borrower is given by

\[ \tilde{n}(A) = 1 \]  
\[ \tilde{r}(A) = \frac{1}{T} \ln \left[ \frac{1}{p} e^{\frac{T}{p}} + \frac{p - 1}{p} V(A) \right] \]

for the worse borrower, \( A \), and

\[ \tilde{n}(\overline{A}) = \begin{cases} 
  n_{E'} & \text{if } C(E', \overline{A}) < C(E'', \overline{A}) \\
  n_{E''} & \text{if } C(E'', \overline{A}) < C(E', \overline{A}) \\
  \frac{\ln (\Omega+1)}{\ln p} & \text{if } \frac{\ln (\Omega+1)}{\ln p} \in \mathbb{N}
\end{cases} \]  
\[ \tilde{r}(\overline{A}) = \begin{cases} 
  r_{E'} & \text{if } C(E', \overline{A}) < C(E'', \overline{A}) \\
  r_{E''} & \text{if } C(E'', \overline{A}) < C(E', \overline{A}) \\
  \frac{1}{T} \ln [\Omega+1] \ln \left[ \frac{p - 1}{p} e^{\frac{T}{p}} + \frac{p - 1}{p} V(\overline{A}) + \frac{\Omega+1-p}{\Omega+1} \right] & \text{if } \frac{\ln (\Omega+1)}{\ln p} \in \mathbb{N}
\end{cases} \]

for the better borrower, \( \overline{A} \).

There does not exist a simple rule or condition to eliminate any of these cases. In reality however, the difference between these cases is expected to be relatively minor, and their effect on comparative statics is likely to be of similarly low significance. Of further note is the fact that the first of these cases, where \( E' \) is chosen, actually sees lenders earn a positive profit, since \( r_{E'} > r_0 \). A deviation from this contract to one with zero profit would however fail to attract any borrowers of this type, since its cost to the borrower would be greater.

Following the specification of the equilibrium menu, it becomes clear that it illustrates a term structure for the interest rates charged on loans in the interbank market. The complete menu specifies a precise interest rate for each maturity, and each type of borrower chooses one such contract. Indeed, given continuous contract dimensions and a continuous range of types, this yield curve would be constructed as a continuous function as well, albeit using a somewhat different methodology (Myerson 1979). Figure 4 shows this relationship as a yield curve, plotting interest rates against maturity \( \tau = \frac{T}{n} \) for the two type case. When
\[ \frac{\Delta s}{\Delta n} < 0, \] one observes the usual upward sloping pattern for a yield curve. Of course when this inequality is reversed, \( r_E(A) > r^*(A) \) and hence the yield curve becomes inverted.

![Figure 4: Interbank Market Yield Curve with Two Borrower Types](image)

This market equilibrium does not exhibit rationing, since every borrower can in fact secure \( L \) in reserves, owing to the previous assumption that lenders have sufficient funds to service the market.

As a final note on this equilibrium, it is possible to conclude that even with screening, the impact of asymmetric information of the market is a pareto decrement. Note by comparing Figures 2 and 3 that the perfect information contract \((r^*(A), 1)\) must always lie below the isocost curve associated with the screening contract \((r_E, n_E)\).

Since lenders do not earn an economic profit from \((r_E, n_E)\), it is therefore possible to conclude that the participants in the market have suffered an aggregate decrease in welfare. This result is unlikely to be mitigated by the small potentially positive profits earned when \( n \notin N \).
4 Analysis of a Crisis

4.1 Direction of Effects

In order to use the model to produce results and outline the likely effects of changes in the environment, such as greater uncertainty about counterparty type or changes in central bank policy tools, comparative statics may be used. In effect, this involves the use of derivatives to describe the change to the contract terms of the equilibrium menu when key parameters change. It should be noted that for simplicity the “ideal” equilibrium point $E$ is used for these comparative statics. The observed direction of any changes should not be affected by this however, provided the initial shock is large enough, which is a reasonable assumption within the context of a crisis.

The following table summarises the results obtained by this model, subject to the assumptions and properties outlined above[^1].

<table>
<thead>
<tr>
<th>Variable</th>
<th>Nature of Change</th>
<th>$\Delta n_E(A)$</th>
<th>$\Delta r_E(A)$</th>
<th>$\Delta n_E(A)$</th>
<th>$\Delta r_E(A)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>The probability of repayment falls as the crisis intensifies</td>
<td>None</td>
<td>Increase</td>
<td>Increase</td>
<td>Increase</td>
</tr>
<tr>
<td>$A$</td>
<td>The value of the best borrower type’s assets falls</td>
<td>None</td>
<td>None</td>
<td>Decrease</td>
<td>Decrease</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>The value of the worst borrower type’s assets falls</td>
<td>None</td>
<td>Increase</td>
<td>Decrease</td>
<td>Decrease</td>
</tr>
<tr>
<td>$r$</td>
<td>The central bank reduces its interest payment on reserves as it cuts the cash rate target</td>
<td>None</td>
<td>Decrease</td>
<td>None</td>
<td>Decrease, but always by less than $r_E(A)$</td>
</tr>
</tbody>
</table>

[^1]: Note that any ambiguities are excluded when $\frac{\partial D}{\partial A} < \frac{(1-p)(\frac{\partial V}{\partial A} - \frac{\partial D}{\partial A})(\frac{\partial V}{\partial A} - \frac{\partial D}{\partial A})}{\alpha + (p-1)(\frac{\partial V}{\partial A} - \frac{\partial D}{\partial A})}$ and $\left| \frac{1}{\ln(\Omega+1)} \ln \left[ \frac{\ln(\Omega+1)}{\Omega+1} \right] \right| > \left| \frac{\ln(\Omega+1)}{\Omega+1} \right| \ln \left( \frac{\Omega+1}{\Omega+1} \right) \left( \frac{(1-p)\Omega^2}{\Omega^2} \right) \left( \frac{(1-p)(\Omega^2-1)}{\Omega^2} \right)$ |
4.2 Interpretation and Significance

The findings of this model have interesting implications on transparency regulations of financial institutions participating in interbank markets. In effect, the source of asymmetric information in this model is the fact that the true value of assets held by banks on their books is only known to them. Other banks, and indeed any individual outside the organisation, relies on balance sheets and similar accounting statements which are published on a quarterly basis.

When the beginning of the game at $t = 0$ is at a point in time at which the most recent financial statements are some months old, the information available to lenders used to judge borrower quality is relatively poor. As a result, their ability to distinguish types a priori is particularly poor, and screening mechanisms are more likely to be important. In terms of this model, better information available to borrowers as a result of accurate financial reporting should bring about an equilibrium that is closer in nature to the perfect information scenario. This is significant, since the move of $\bar{A}$ away from a perfect information contract towards that offered as part of the screening equilibrium is associated with an increase in cost. As such, the introduction of information asymmetry constitutes a pareto decrement.

For regulators, this suggests that disclosure standards for banks should seek to keep balance sheet information as up to date as possible at all times, including of trading books and any other type of asset that could potentially be sold to repay loans that can no longer be repaid through cash flows.

Furthermore, the model’s finding with regards to central bank policy tools has direct relevance to policymakers. Simple models of monetary policy implementation suggest that a predictable response of interbank markets allows central banks to change short term cash rates by simply varying aggregate reserves. This model suggests that this view may be too simple. Firstly, even when aggregate reserves remain unchanged and the central bank undertakes no policy action, a change in variables like $p$, $\bar{A}$ or $\bar{A}$ can lead to the rates at the short end of the yield curve varying to a potentially significant extent. This mirrors the difficulties faced by central banks during the crisis.
Furthermore, another urgent concern then was the transmission of changes in these overnight rates to longer term rates and ultimately the real economy. In this respect, monetary policy relies on a relatively predictable term structure of interest rates. However, existing ideas of central banking do not take into account the effect of asset price volatility on the implementation of monetary policy itself. The model outlined in this thesis suggests that this may be a significant shortcoming. When the support of asset price distributions suddenly widens due to a decrease of the worst case value realisation, this section argued that spreads between longer and shorter maturities could widen to a potentially significant extent. Such effects violate the requirement of a predictable term structure of interest rates, and may compromise the transmission of monetary impulses through the banking system.

This is of course not an argument for the targeting of asset prices by monetary authorities tout court, and this paper does not offer suggestions on the means of promoting asset price stability. It does however offer a warning that “ideal”, textbook inflation targeting or similar monetary policy designed with a simple “one instrument - one target” rule in mind could have unexpected effects if asset price volatility is excessive.

5 Conclusion

This paper outlined an alternative derivation of the term structure of unsecured interbank interest rates, motivated by the events of the financial crisis in 2007 and 2008. During this crisis, significant doubts about counterparty quality emerged where previously credit risks had been considered so low as to be negligible. Previously unheard of spreads between overnight interest rates and those of longer maturities, such as 3-month LIBOR, emerged and appeared unaffected by central bank changes to cash rate targets.

This paper explains these phenomena by modelling interbank markets as screening games, in which the design of debt contracts includes both an interest rate and maturity dimension. Using these two dimensions, lending banks can design contracts to attract specific types of borrowers, allowing markets to clear and an equilibrium to emerge within an imperfect information environment. This equilibrium then specifies what amounts to a yield curve for interbank markets,
which is shown to be affected by changes in the uncertainty about counter-party repayment probabilities and the recovery value arising from assets held on balance sheets. Interestingly, this setting also predicts that the sensitivity of longer maturity interest rates to changes in central bank policy rates decreases as uncertainty becomes more serious. As such, in case of sudden asset price falls, monetary policy transmission may be compromised precisely when policymakers may look to inject monetary stimuli into the real economy.

Given that this model is a completely novel approach to the modelling of interbank markets, and that the use of maturity as a contract dimension has rarely been explored in the screening literature, there are numerous extensions and modifications that could form the base of future research. In particular modelling the market for deposits and the use of multi-unit discriminatory auction theory to model central bank policy interventions could be suggested, although these would add considerable complexity to the paper.

In conclusion, this model can only be seen as a first step towards a more thorough understanding of the applications of information economics in general and screening game theory in particular to interbank markets. Its key findings are relevant to both theorists and policymakers alike. Price volatility in asset classes that form significant parts of bank balance sheets, and indeed a lack of transparency in financial institutions, can make monetary policy implementation more complex and hence make rapid and accurate monetary policy responses to economic shocks more difficult.
References


Basov, S. (2005), Multidimensional Screening, Berlin: Springer Verlag.


Appendix

Appendix I - Derivation of the General Cost Function

Recall that for \( n = 3 \), the cost function was given by

\[
C(r, 3, A) = \alpha L + p^3 \left[ 2(e^{rT_3} - 1 + \alpha)L + e^{rT_3}L \right] + p^2(1 - p) \left[ 2(e^{rT_3} - 1 + \alpha)L + D(A) \right] + p(1 - p) \left[ (e^{rT_3} - 1 + \alpha)L + D(A) \right] + (1 - p)D(A)
\]

Generalising this functional form to an unspecified \( n \) yields:

\[
C(r, n, A) = \alpha L + p^n \left[ (n - 1)(e^{rT_n} - 1 + \alpha)L + e^{rT_n}L \right] + (1 - p) \left( \sum_{i=0}^{n-1} ip^i \right) \left[ (e^{rT_n} - 1 + \alpha)L \right] + (1 - p) \left( \sum_{i=0}^{n-1} p^i \right) D(A)
\]

Expanding the geometric series yields

\[
\sum_{i=0}^{n-1} ip^i = \frac{p^n(np - n - p) + p}{(p - 1)^2}
\]

and further

\[
\sum_{i=0}^{n-1} p^i = \frac{p^n - 1}{p - 1}
\]

Substituting these results into the cost function:

\[
C(r, n, A) = \alpha L + p^n \left[ (n - 1)(e^{rT_n} - 1 + \alpha)L + e^{rT_n}L \right] - \frac{p^n(np - n - p) + p}{p - 1} \left[ (e^{rT_n} - 1 + \alpha)L \right] - (p^n - 1)D(A)
\]
It can be seen that the expected cost is therefore a function of the cost in case there is no default at all during $T$, which occurs with probability $p^n$, the possible failure to make one or more interest payments, occurring with $\frac{p^n(np - n - p) + p}{p - 1}$ and the probability of defaulting during the entirety of $T$, given by $1 - p^n$.

Finally, rearranging this equation returns the following expression:

$$C(r, n, A) = \frac{p^n - 1}{p - 1} \alpha L + \frac{p(p^n - 1)}{p - 1} e^{r T}(\frac{p^n - p}{p - 1}) L - (p^n - 1)D(A)$$
Appendix II - Derivation of Two-Type Screening Contract offered to \( \bar{A} \)

We begin by observing in Figure 3 that the intersection of \( r_0(n, \bar{A}) \) and \( s_h(n, \bar{A}) \) minimises borrower cost subject to non-negative profits for the lender. Keeping in mind that \( h = C(r^*(\bar{A}), 1, A) = \alpha L + e^{rT}L + (p-1)(V(\bar{A}) - D(\bar{A})) \), we can therefore determine the interest rate and number of roll overs as follows:

\[
\begin{align*}
   r_0(n, \bar{A}) &= s_h(n, \bar{A}) \\
   \frac{p-1}{p(p^n-1)} e^{rT} + \frac{(p-1) V(\bar{A})}{p} - \frac{h}{p} &= \frac{p-1}{p(p^n-1)} \frac{h}{L} + \frac{p-1}{p} \frac{D(\bar{A})}{L} - \frac{\alpha}{p} \\
   \frac{p-1}{p(p^n-1)} \left( \frac{h}{L} - e^{rT} \right) &= \frac{p-1}{p} \left( \frac{V(\bar{A})}{L} - \frac{D(\bar{A})}{L} \right) + \frac{\alpha}{p} \\
   p^n - 1 &= \frac{V(\bar{A})}{L} - \frac{D(\bar{A})}{L} + \frac{\alpha}{p-1} \\
   &= \frac{\alpha + e^{rT} + (p-1)(\frac{V(\bar{A})}{L} - \frac{D(\bar{A})}{L}) - e^{rT}}{\frac{V(\bar{A})}{L} - \frac{D(\bar{A})}{L} + \frac{\alpha}{p-1}} \\
   &= \frac{\alpha + (p-1)(\frac{V(\bar{A})}{L} - \frac{D(\bar{A})}{L})}{\alpha + \frac{(p-1)(\frac{V(\bar{A})}{L} - \frac{D(\bar{A})}{L})}{\alpha + \frac{\alpha}{p-1}}} \\
   &= \Omega \\
   n_E &= \frac{\ln(\Omega + 1)}{\ln p}
\end{align*}
\]

The associated zero-profit interest rate is given by:

\[
\begin{align*}
   r_E &= \frac{n_E}{T} \ln \left[ \frac{p-1}{p(p^n-1)} e^{rT} + \frac{(p-1) V(\bar{A})}{p} - \frac{p^{nE} - p}{p(p^n-1)} \right] \\
   &= \frac{1}{T} \ln \left[ \frac{p-1}{p(\Omega + 1 - 1)} e^{rT} + \frac{(p-1) V(\bar{A})}{p} - \frac{\Omega + 1 - p}{p(\Omega + 1 - 1)} \right] \\
   &= \frac{1}{T} \ln \Omega + 1 \ln \left[ \frac{p-1}{p} e^{rT} + \frac{p-1 V(\bar{A})}{p} - \frac{\Omega + 1 - p}{p\Omega} \right]
\end{align*}
\]