Summary of

Analysis of Premium Liabilities for Australian Lines of Business

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Background

Following the introduction of APRA’s new prudential regulatory framework in July 2002, insurers are now required to value their unexpired risk prospectively. A reserve referred to as Premium Liability values the estimated losses that are expected to arise in the future from business which have already been underwritten. Under General Prudential Standard 210 (Australian Prudential Regulation Authority 2002 (a)) general insurers are required to report the central estimates and risk margins associated with their Premium Liability, and Outstanding Claims Liability for all lines of business underwritten.¹

As Premium Liability is a liability which has not yet occurred, the estimation of its value will require some form of modelling. This future liability is dependent on two main variables: the frequencies of future claims and future claim costs for the policies currently in force. Both variables are random in nature, so there is a high level of uncertainty associated with their prediction. Currently, there is limited literature available exploring the statistical nature of Premium Liability.

Aim and Significance of Research

This research paper supplements the limited pool of current literature by presenting a theoretical model that can be adopted to examine the statistical behaviour of future claims liabilities. A simple model is constructed and used to estimate the first two central moments of both Premium Liability and Outstanding Claims Liability. The results from the model will be used to compare the variability of Outstanding Claims Liability to the variability of Premium Liability for long-tailed and short tailed lines of Australian business. This paper will argue that the co-efficients of variation of Outstanding Claims Liability and Premium Liability differ and the magnitude of the difference depends on whether the line of business is long or short-tailed.

The significance of this result is that the ratio between the volatility associated with the estimation of Premium Liability to the volatility associated with the estimation of Outstanding Claims Liability should not be constant for all lines of business. The research results conclude that this ratio is larger for long-tailed lines of business. This is a consequence of the averaging or pooling effects across the independent accident years. The pooling effect plays a more prominent role in reducing the variability of the Outstanding Claims Liability of longer-tailed lines of business.

¹ Premium Liability refers to an insurer’s claim liabilities arising from future claimable events for policies which are currently in force. Outstanding Claims Liability refers to an insurer’s claim liabilities for claimable events which have already occurred, but have not been notified to the insurer.

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Research Approach

Model for calculating future claim payments:

The model adopted requires the input of run-off claims data and is designed to calculate the mean and variance for total claims payments in any single accident year. Using this approach, the model can be applied to a range of different situations and allows further assumptions to be incorporated on a case-by-case basis. It has been assumed that the volatility of future claim payments in each accident year is affected by only two independent sources of uncertainty:

1. The number of claims that will occur, which will be denoted by the random variable \( N^i \) and;
2. The average size of future claim payments, which will be denoted by the random variable \( X^i \).

The model is constructed as follows where the random variable \( S^i \) is used to denote the total dollar amount of payments made for claims incurred during accident year \( i \).

\[
\text{Total Claim Payments} (S^i) = \text{Total number of claims} (N^i) \times \text{Average cost per claim} (X^i)
\]

\[
\downarrow \quad \downarrow
\]

\[
N(\mu_1, \sigma_1) \quad N(\mu_2, \sigma_2)
\]

Expected value = \( \mu_1\mu_2\)
Variance \( \rightarrow \) derived by simulation

The model is based on the assumption that the total claim payments made in any accident year is equal to the product between the total number of claims which occurred and the average claim payments made in respect of those claims.

For simplicity’s sake, the model assumes that the behaviour of the two individual random variables follow a normal distribution. Using run-off data collected from reported claims, the values of \( N^i \) and \( X^i \) can be calculated using the Chain Ladder Method for the previous number of accident years for which data has been collected. A trend line can be fitted across these data points and then projected forward to obtain the future expected values of \( N^i \) and \( X^i \). The volatility can be measured by calculating the deviation of
the data points from the trend line. $N^i$ and $X^i$ can now be modelled as two independent normal random variables with a mean and variance estimated by the past data. An example of this analysis is seen below in Figure 1.

![Motor Vehicle](attachment:Motor_Vehicle.png)

**Figure 1**

This method will be adopted to find the mean and variance of Premium Liabilities and will require the estimation of $E(N^i)$, $V(N^i)$, $E(X^i)$, and $V(X^i)$ for the subsequent accident year.

As $N^i$ and $X^i$ are assumed to be independent, it follows that:

$$E(S^i) = E(N^i)E(X^i)$$

And,

$$V(S^i) = V(N^iX^i)$$

Values for the mean and variance of $N^i$ and $X^i$ have already been estimated by the model. The assumption of normality allows values of $N^i$ and $X^i$ to be generated by simulation and multiplied with each other to produce simulated values of $S^i$. The variance of a large sample of simulated values is used as an approximation to the variance of $S^i$. Assuming that individual accident years are independent and identically distributed, the model can also be used to estimate the mean and variance of the corresponding Outstanding Claims Liability. This allows for comparison between the co-
coefficients of variation of Premium Liability and Outstanding Claims Liability for different lines of business.

Data

The Insurance and Superannuation Commission, many of the functions of which now form part of APRA, released detailed claims analysis data for general insurance lines of business over the period 1983 - 1996 covering three lines of business: motor vehicle, public liability, and compulsory third party (Insurance and Superannuation Commission 1983 - 1996).

Premium Liability

Total Number of Claims - \( N^i \)

The trendline is a function of the accident years and is representative of the expected total number of claims reported in each accident year. It is expressed as \( Y = aX + b \), where \( Y \) is the total number of claims reported and \( X \) is the accident year. Once the values of \( a \) and \( b \) are calculated, the trendline can be used to forecast the expected value of \( N^i \) for the subsequent accident year. In the model used for this research, \( X \) is replaced by the year 1997 (insurance data is only provided up to 1996). This value is the expected total number of claims in the following accident year - \( E(N^{1997}) \).

As the data collected represents the aggregate experience of the private insurance sector, it is reasonable to assume that all un-systematic risks have been diversified away, so that the deviations from the trendline (the mean) correspond only to the systematic risk or the inherent uncertainty associated with estimation of the liability.

On the assumption that the individual accident years are independent, the variance of the total number of claims for each individual accident year can be defined as:

\[
V(N^i) = \frac{\left( \sum_{i=1}^{10} \text{deviation of data point from the trendline} \right)^2}{14 - 2}
\]

for \( i = 1983, 1984, \ldots, 1996 \)

Two degrees of freedom are lost in the denominator due to the estimation of the two parameters for \( E(N^i) \) and \( V(N^i) \).

Average Payment per Claim - \( X^i \)
The average payment per claim in accident year $i$ is defined as:

$$X^i = \frac{\text{total claim payments for accident year } i}{\text{total number of claims settled in accident year } i} = \frac{P^i}{\delta^i}$$

The claim payments are recorded as nominal amounts. In order to consistently compare the payments made at different times, the claim payments are adjusted to a stable currency. Average Weekly Earnings (AWE) is used as the inflation index. Payments are assumed to be made in the middle of each development year. The AWE index is preferred over the Consumer Price Index, the typical measure of inflation, due to the large proportion of the claim costs being wages related. $E(X^i)$ and $V(X^i)$ for average payment per claim are calculated in the same manner using process which calculated $E(N^i)$ and $V(N^i)$ for total number of claims in the previous section.

**Outstanding Claims Liability**

While Premium Liability estimates the claim payments with respect to a single accident year in the future, Outstanding Claims Liability estimates future claim payments for claims which occurred over a number of earlier accident years. For each of these accident years, only a portion of the total claim payments remains outstanding; the other portion has already been reported at the time of valuation. The number of accident years that the Outstanding Claims Liability spans is dependent on the nature of the liabilities. For long-tailed lines of business such as public indemnity, this number is quite high (typically in excess of 7 years) and for short-tailed claims such as motor vehicle, this number is much smaller (1 to 2 years).

Assuming that the individual accident years are independent, the variability of the Outstanding Claims Liability will be reduced comparatively to the corresponding Premium Liability due to the effects of pooling across the independent accident years. It can be shown that this averaging affect becomes more dominant when the Outstanding Claims Liability contributes to a bigger proportion of the total claim payments or when the Outstanding Claims Liability spans over a higher number of accident years. In other words, there is a larger comparative reduction in volatility between Outstanding Claims Liability and Premium Liability for longer-tailed lines of business compared to short-tailed business.

Available data covers 14 accident years and 10 development years. Based on this data, the ratios between the co-efficients of variation between Premium Liability and Outstanding Claims Liability are compared for short tailed and long tailed lines of business using the following formulas.
Let $S^i$ be defined as the total claim payments of accident year $i$, which is a random variable. Let $f_{i,j}$ be defined as the proportion of $S^i$ that is paid in development year $j$. Finally $F_i$ is defined as the sum of $f_{i_{12-j}}$, $f_{i_{31-j}}$, ... , and $f_{i_{10}}$ (i.e. $F_i = \sum_{j=12-i}^{10} f_{i,j} = \text{outstanding proportion}$).

It can be seen that the total Outstanding Claims Liability, $OS$ at a point in time after the 14 accident years is equal to $\sum_{i=2}^{14} F_i S^i$ and its coefficient of variation is:

$$CV(OS) = \sqrt{\frac{\text{Var}(OS)}{E(OS)}} = \sqrt{\frac{\sum_{i=2}^{14} F_i^2 \text{Var}(S^i)}{\sum_{i=2}^{14} F_i E(S^i)}}$$

Let $P$ be the Premium Liability, the coefficient of variation at the same time is:

$$CV(P) = \sqrt{\frac{\text{Var}(P)}{E(P)}}$$

Assuming that for each accident year $E(S^i) = \mu$ and $\text{Var}(S^i) = \sigma^2$, the results can be derived as follows:

$$CV(P) = \frac{\sigma}{\mu} \text{ and } CV(OS) = \frac{\sigma}{\mu} \sqrt{\frac{\sum_{i=2}^{14} F_i^2}{\sum_{i=2}^{14} F_i}}$$

It then follows that:

$$CV(P) > CV(OS)$$

**Results**

The three lines of business under consideration are compulsory third party, public liability and motor vehicle. The first two lines are considered to produce long-tailed claims while the motor vehicle business is short-tailed in nature. The Chain Ladder method (Buchanan, Hart et al. 1996) is adopted to estimate the Outstanding Claims Liability and the following table presents the results.

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The results clearly show that the averaging effects are much greater for the long-tailed businesses. In other words, the ratio between the co-efficients of variation of Premium Liability to that of Outstanding Claims Liability for long-tailed liabilities is greater than the ratio for short-tailed risks.

### Implications for Prudential Supervision

The Risk Capital Factors are prescribed by APRA for the calculation of the Insurance Risk Capital Charge\(^2\) (Australian Prudential Regulation Authority 2002 (b)). The Risk Capital Charge needs to reflect the inherent uncertainty associated with the estimation of the underlying liability and should therefore be proportional to the co-efficient of variation of the future liability. The results suggest that the Premiums Liability Risk Capital Factors prescribed by APRA to calculate the Insurance Risk Capital Charge are inadequate for the more volatile lines of business. The Premiums Liability Risk Capital Factors set by APRA are 150% of the corresponding Outstanding Claims Liability Risk Capital Factors for all lines of business. This constant 50% loading would only be reasonable if the variability of the Premium Liability exceeds the variability of the Outstanding Claims Liability by the same amount for all lines of business. This is not the case as the results show that the co-efficient of variation of the Premium Liabilities exceeds the co-efficient of variation of Outstanding Claims Liability by a much greater amount for the longer-tailed risks.

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\(^2\) The Insurance Risk Capital Charge is a margin held to buffer the risk that the actual value of the liability is greater than the expected value calculated by the insurer.

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Conclusion

The rapid advancement of technology has allowed for the construction of more intricate and realistic models. Although these models can produce results with higher degrees of accuracy, they are often much more complex and data intensive. If the right assumptions are not used or the data is unreliable, these models may simply produce more statistical noise. This paper has provided a starting point to Premium Liability modelling by introducing a basic model with relatively few assumptions.

This paper has also highlighted a relationship between the co-efficients of variation for Outstanding Claims Liability and Premium Liability. The results show that the co-efficient of variation of the Premiums Liabilities for some lines of business is inaccurately reflected by Premiums Liability Risk Capital Factors which all have a constant 50% loading. This is a result of the averaging or pooling effects across the independent accident years, and plays a more prominent role in long-tailed lines of business.

The prediction of future claims liability is highly speculative by nature and this paper has only provided an introduction to its estimation. It is hoped that further research will construct and test more sophisticated models which incorporate additional assumptions and parameters. Such analysis will improve our understanding and further our ability to predict and manage the random variation of these liabilities.
References


Australian Prudential Regulation Authority (2002(b)). Guidance Note GGN 110.3 - Insurance Risk Capital Charge.
